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Spatially Varied Subcritical and Supercritical Flow in Gullies

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SPATIALLY VARIED SUBCRITICAL AND SUPERCRITICAL FLOW IN GULLIES

By

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ABSTRACT

The objective of this study was to investigate the phenomena of subcritical and supercritical spatially varied flow in rectangular expansions such as that in an erosion gully. Based on momentum and continuity principles, equations were developed to predict such flow phenomena. Computations of the water surface profiles for subcritical spatially varied flow were carried out by applying direct step numerical calculations. Because of the presence of standing waves, the method of characteristics was employed in the analytical analysis of the supercritical flow phenomena. Of primary importance in this study is the effect of varying amounts of lateral inflow rates. Other parameters varied during the experimental tests were the slope of the channel and the main channel discharge. Comparison of the analytical and experimental results indicated that for both subcritical and supercritical spatially varied flow, water surface profiles can be predicted with good accuracy.

KEY WORDS: open channel flow, transition flow, gullies flow, subcritical flow, supercritical flow, spatially varied flow, standing waves, method of characteristics

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CHAPTER I

INTRODUCTION

Spatially varied or discontinuous flow can be defined as flow in which the discharge varies in the flow direction. There are basically two forms of spatially varied flow: flow with decreasing discharge and flow with increasing discharge.

Spatially varied flow with increasing discharge occurs in such places as side-channel spillways of dams; wash-water troughs in water and sewage treatment plants; and overland flow into rain gutters or gullies. In the case of a rain gully, not only is there increasing discharge, but the width also increases as the flow moves downstream.

A channel, whether artificial or natural, seldom has a straight, horizontal alignment. Most channels may vary in cross section as well as slope. Channel expansion occurs frequently at places where the flow emerges from a closed conduit, sluice gate, spillway or steep chute. Thus, the ability to predict accurately the profile of spatially varied flow in a channel expansion is of considerable importance.

REVIEW OF PREVIOUS WORK

One of the first substantially correct differential equations describing spatially varied flow was established by Hinds (1) in 1926. His study dealt with the design of side-channel spillways. In his analysis, Hinds neglected the momentum due to the lateral inflow and the effect of friction. Favre (2) included a friction term in a similar study. In 1940, Camp (3) investigated rectangular channels with uniform lateral inflow. His main objective was analyzing lateral spillway channels as found in water and sewage treatment plants. Camp's equation is the same as Hinds' except for the inclusion of a friction term. Camp expressed the frictional resistance in terms of the Weisbach-Darcy friction factor, f. Li (4) obtained solutions for prismatic channels of various cross sections and slopes. He described the effect of friction in terms of the Chezy coefficient, C.

Yen and Wenzel (5) developed equations for steady spatially varied flow. These equations resulted from both momentum and energy approaches. In their study only channels with straight, horizontal alignment and longitudinally uniform cross section were considered.

A study dealing with supercritical flow passing through transitions was conducted by Ippen (6) in 1951. When the boundary alignment along the side walls is gradual,

small changes in depth resulted. For this reason Ippen neglected vertical acceleration, boundary - layer influences and nonuniformities in the velocity and pressure distributions. An analysis similar to Ippen's was made by Harrison (7). His analysis applied to the design of channels with a trapezoidal cross section.

The energy loss in a sudden expansion can be reduced by gradually enlarging the channel section or decreasing the angle of divergence (8). Rouse, Bhoota and Hsu (9) investigated high-velocity flow in abrupt and gradual expansions. The diverging section must be gradual enough to avoid separation at the entrance to the section. In order to eliminate a long diverging section, they suggested a continuous change in curvature which would allow the section to be shorter and reduce the chances of separation. Studies by Hom-ma and Shima (10) further indicate that separation may occur if an expansion is made to diverge too rapidly.

Standing waves are present in both subcritical and supercritical flow. Supercritical flow presents a problem because of the surface waves that appear as a consequence of the geometry of the lateral boundaries. In a symposium on high-velocity flow, Knapp (11) pointed out the presence of standing waves created in curved channels. Bagge and Herbich (12) analyzed, by using the method of characteristics, supercritical flow in a rectangular expansion.

They assumed frictionless flow and hydrostatic pressure distribution. Later, Herbich and Walsh (13) tested the equations of Bagge and Herbich for validity. They found the predicted profiles were generally lower than those measured, and attributed this to the neglection of friction. The United States Soil Conservation Service (14) has also made tests on supercritical flow through expansions in conjunction with the SAF stilling basin. The tests were made on transitions with straight flaring side walls and one percent channel slope.

SCOPE OF THE PRESENT STUDY

In this study subcritical and supercritical spatially varied transitional flow in a rectangular expansion is investigated. The lateral inflow is in the form of uniform sheet flow along the entire length of the diverging test section. Of primary importance are the effects of varying amounts of lateral inflow. The experimental results for both types of flow are compared with analytical solutions. Because of the high energy loss at the point of impact of the lateral inflow and the main flow, the momentum approach is utilized in the analysis rather than the energy approach.

CHAPTER II

ANALYTICAL ANALYSIS

DYNAMIC EQUATION FOR SUBCRITICAL SPATIALLY VARIED FLOW

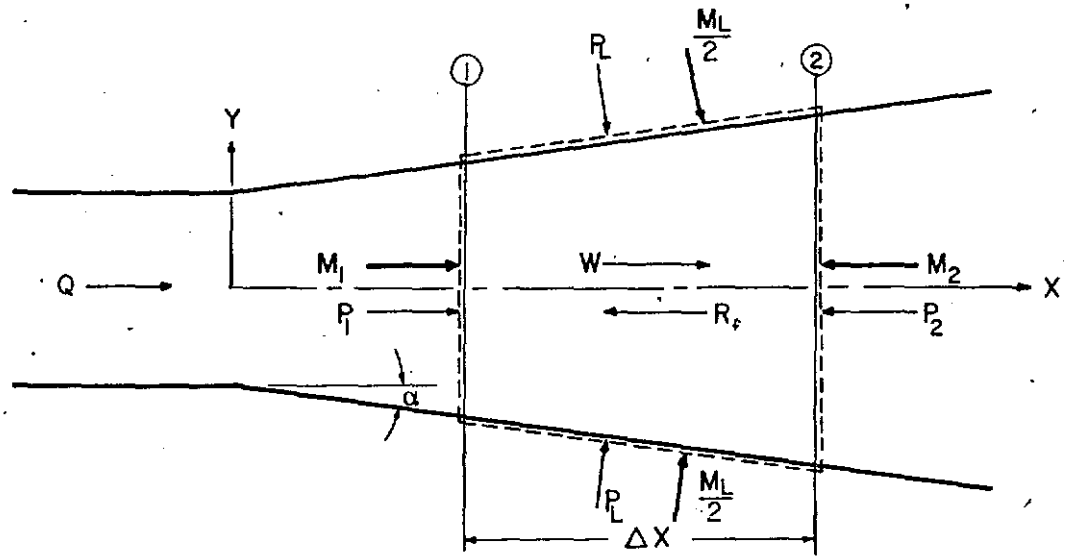
By use of momentum principles, an expression for the change in water depth with respect to distance is determined.

The depth, d, of the flow section is measured normal to the channel floor, which makes an angle θ with the horizontal. Therefore, the channel slope, S_o, equals the tangent of the angle θ. The lateral inflow, q_*, is defined as the uniform volumetric inflow per unit time per unit length along the sidewall. At the point where the lateral inflow joins the main flow, q_* has a velocity U which makes an angle ϕ with the flow direction. In the case of vertical lateral inflow, ϕ equals $(\frac{\pi}{2} - \theta)$.

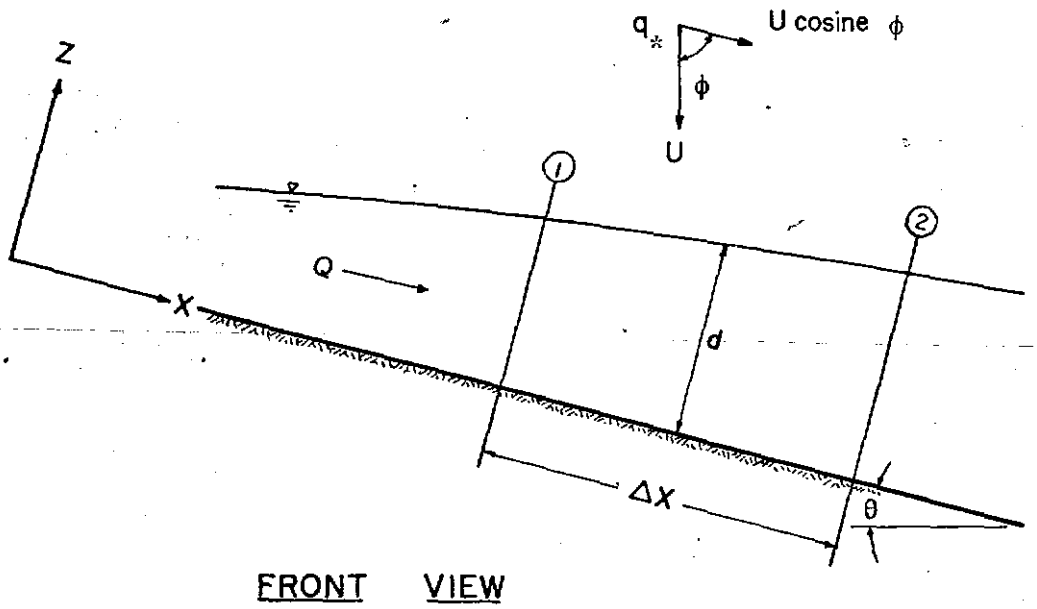
For the element of fluid between sections 1 and 2, as shown in Fig. 1 the summation of forces in the x-direction, including dynamic forces, equals zero. Mathematically, this can be expressed as:

$$M_1 + P_1 + 2P_{LX} + M_{LX} + W - R_f - M_2 - P_2 = 0 \qquad (1)$$

where M_1 and M_2 are the momenta at sections 1 and 2 (per sec) respectively, P_1 and P_2 are the pressure forces at sections



TOP VIEW



FRONT VIEW

Fig. 1 - Definition Sketches for Subcritical Flow

1 and 2 respectively, P_{LX} is the x-component of the pressure force acting on the sidewall, M_{LX} is the x-direction momentum flux due to the lateral inflow, W is the force due to the weight of the fluid and R_f is the boundary resistance.

The change in water depth between sections 1 and 2 is $\Delta d = d_2 - d_1$ and the corresponding distance in the x-direction is $\Delta x = x_2 - x_1$, where x_1 and x_2 are measured along the centerline from the beginning of the diverging test section.

Substituting the values of the forces, Eq. 1 becomes:

$$\rho\beta V_1^2 B_1 d_1 + K\gamma B_1 d_1^2 \cos\theta + 2K\gamma \left(\frac{d_1 + d_2}{2}\right)^2 \Delta x \cos\theta \sin\alpha$$

$$+ \rho q_* \Delta x U \cos\phi \cos\alpha + \left(\frac{A_1 + A_2}{2}\right) \Delta x \gamma \sin\theta$$

$$- \frac{1}{2}\left(B_1 + 2d_1 + B_2 + 2d_2\right) \Delta x \tau - \rho\beta V_2^2 B_2 d_2$$

$$- K\gamma B_2 d_2^2 \cos\theta = 0 \qquad (2)$$

in which ρ is the mass density of the fluid, β is the momentum flux correction factor, V_1 and V_2 are the mean velocities at sections 1 and 2 respectively, B_1 and B_2 are the widths in the y-direction at sections 1 and 2 respectively, K is the pressure correction factor, γ is the specific weight of the fluid, α is the divergent angle, A_1 and A_2 are the cross sectional areas at sections 1 and 2

respectively and τ is the x-component boundary shear stress.

Since linear expansion is considered:

$$B_2 = B_1 + 2\Delta x \tan \alpha \tag{3}$$

Neglecting all higher order differentials and combining like terms in Eq. 2 will yield:

$$(- \rho\beta V_2^2 B_1 - 2K\gamma B_1 d_1 \cos\theta)\ \Delta d$$

$$+ (\rho q_* U \cos\phi \cos\alpha + B_1 d_1 \gamma \sin\theta + 2K\gamma d_1^2 \cos\theta\ (\sin\alpha - \tan\alpha)$$

$$- 2\rho\beta V_2^2 d_1 \tan\alpha - B_1\tau - 2d_1\tau)\ \Delta x$$

$$+ \rho\beta B_1 d_1 (V_1^2 - V_2^2) = 0 \tag{4}$$

The continuity equation can be written as:

$$Q_2 = A_2 V_2 = Q_1 + q_* \Delta x \cos\phi \cos\alpha \tag{5}$$

with

$$A_2 = B_2 d_2 = B_1 d_1 + B_1 \Delta d + 2d_1 \Delta x \tan\alpha \tag{6}$$

Combining Eqs. 5 and 6 gives:

$$V_2^2 = \frac{Q_2^2}{A_2^2} = \frac{Q_1^2 + 2Q_1 q_* \Delta x \cos\phi \cos\alpha}{B_1^2 d_1^2 + 2B_1^2 d_1 \Delta d + 4B_1 d_1^2 \Delta x \tan\alpha} \tag{7}$$

For

$$V_1^2 = \frac{Q_1^2}{A_1^2} \tag{8}$$

we have

$$V_1^2 - V_2^2 = \frac{2Q_1^2 B_1 \Delta d + 4Q_1^2 d_1 \Delta x \tan \alpha - 2Q_1 A_1 q_* \Delta x \cos\phi\cos\alpha}{A_1^2 (A_1 + 2B_1 \Delta d + 4d_1 \Delta x \tan \alpha)} \quad (9)$$

Substituting Eq. 9 into Eq. 4 yields:

$$\left(\frac{\rho\beta Q_1^2}{B_1 d_1^2} + 2 K \gamma B_1 d_1 \cos \theta \right) A_x \Delta d =$$

$$(\rho q_* U \cos \phi \cos \alpha + B_1 d_1 \gamma \sin \theta + 2K\gamma d_1^2 \cos \theta (\sin \alpha - \tan \alpha)$$

$$- \frac{2 \rho\beta d_1 Q_1^2 \tan \alpha}{B_1^2 d_1^2} - B_1 \tau - 2d_1 \tau) A_x \Delta x +$$

$$\rho\beta B_1 d_1 \left\{ \frac{\Delta d}{A_1^2} (2Q_1^2 B_1) + \frac{\Delta x}{A_1^2} (4Q_1^2 d_1 \tan \alpha - \right.$$

$$\left. 2 Q_1 A_1 q_* \cos \phi \cos \alpha) \right\} \quad (10)$$

where

$$A_x = (A_1 + 2B_1 \Delta d + 4d_1 \Delta x \tan \alpha) \quad (11)$$

By neglecting all of the higher order terms included in the product of $A_x \Delta d$ and $A_x \Delta x$ terms, Eq. 10 can be simplified to yield:

$$\left\{\gamma B_1 d_1 \left(2K \cos \theta - \frac{\beta V_1^2}{g d_1} \right)\right\} \Delta d =$$

$$\left\{\gamma B_1 d_1 \left(\frac{q_* U \cos\phi \cos\alpha}{g B_1 d_1} + \sin\theta + 2K \frac{d_1}{B_1} \cos\theta \, (\sin\alpha - \tan\alpha) \right.\right.$$

$$\left.\left. - \frac{\tau}{\gamma d_1} - \frac{2\tau}{\gamma B_1} - \frac{2\beta V_1 q_* \cos\phi \cos\alpha}{g B_1 d_1} + \frac{2\beta V_1^2 \tan\alpha}{g B_1} \right)\right\} \Delta x \qquad (12)$$

For small angle θ

$$\sin \theta = \tan \theta = S_o \qquad (13)$$

Solving for $\frac{\Delta d}{\Delta x}$ from Eq. 12 gives:

$$\frac{\Delta d}{\Delta x} = \left\{ \frac{q_* U \cos\phi \cos\alpha}{g A_1} + S_o + 2K \frac{d_1}{B_1} \cos\theta \, (\sin\alpha - \tan\alpha) \right.$$

$$\left. - \frac{\tau}{\gamma d_1} - \frac{2\tau}{\gamma B_1} - \frac{2\beta V_1 q_* \cos\phi \cos\alpha}{g A_1} + \frac{2\beta V_1^2 \tan\alpha}{g B_1} \right\} \div$$

$$\left\{ 2 K \cos \theta - \frac{\beta V_1^2}{g d_1} \right\} \qquad (14)$$

which relates the change in water depth to an incremental change in distance in the downstream direction.

THEORETICAL ANALYSIS FOR SUPERCRITICAL SPATIALLY VARIED FLOW

By applying the basic principles of the conservation of momentum and mass, differential equations involving the partial derivatives of the velocity components and water depth with respect to the coordinate system defined in

Figure 2 are derived. Due to the complex nature of the flow system, exact solutions to the equations are not possible. Therefore, a finite difference technique is employed to obtain information concerning the flow properties of the entire flow field. The specific method used in the solution procedure is known as the method of characteristics. For this method to be applicable, irrotational flow is assumed for the entire flow region except in the relatively thin boundary layer in the immediate vicinity of the channel floor.

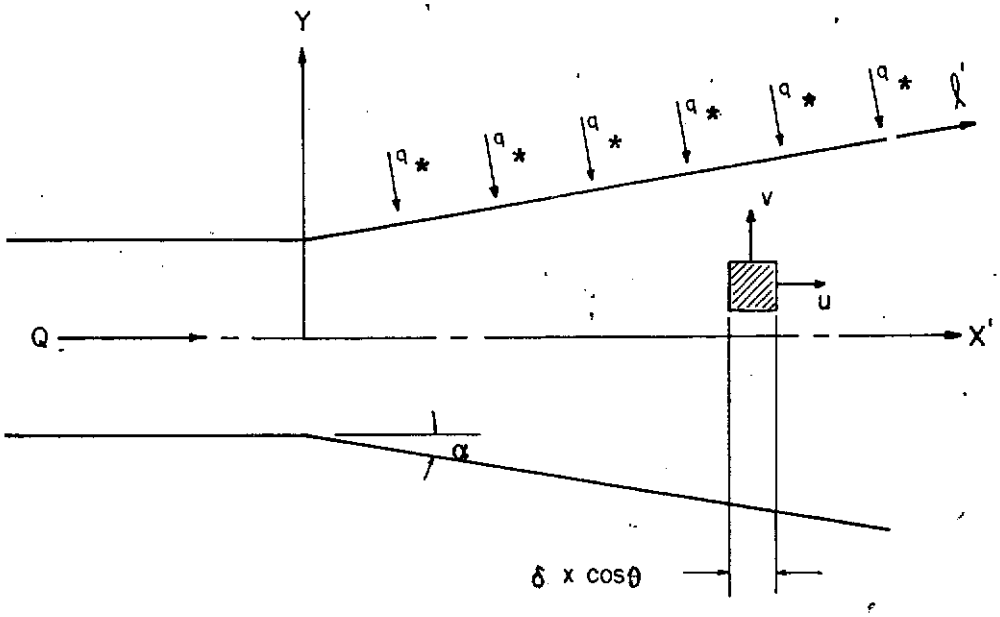
A few other basic assumptions are also made. It is assumed that (1) the pressure distribution is hydrostatic, (2) the velocity distribution in the z-direction is uniform (flow to be irrotational), except in the boundary layer and (3) horizontal momentum component introduced by the lateral inflow, q_*, as it enters the main flow vertically is very small compared with that due to the supercritical main channel flow, and thus, can be neglected.

The continuity equation for all points except at the sidewall is:

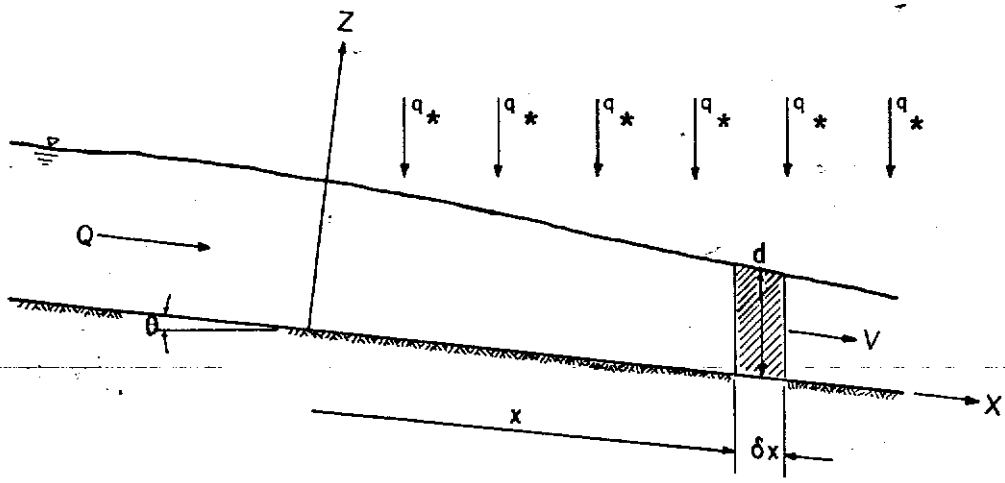
$$d \frac{\partial u}{\partial x} + u \frac{\partial d}{\partial x} + d \frac{\partial v}{\partial y} + v \frac{\partial d}{\partial y} = 0 \qquad (15)$$

at the sidewall, Eq. 15 becomes:

$$d \frac{\partial u}{\partial x} + u \frac{\partial d}{\partial x} + d \frac{\partial v}{\partial y} + v \frac{\partial d}{\partial y} - \frac{q_1 + q_2}{\delta x \delta y} = 0 \qquad (15')$$



TOP VIEW



FRONT VIEW

Fig. 2 – Definition Sketches for Supercritical Flow

where q_1 and q_2 are two portions of the lateral inflow between two adjacent points along the sidewall. The first portion, q_1, represents the lateral inflow over the distance covered by the y-direction projection of the segment between points i-(NP + 1) and i-NP on the sidewall. The distance covered by the projection of line i-NP to i is used to compute the magnitude of q_2. The vertical depth of the flow is d, and α is the divergent angle of the test section. With δx and δy representing incremental changes in distance in the independent x-y coordinate system, u and v are the velocity components in the two directions respectively.

Writing the momentum equation in the x-direction for a fluid element, which does not include the sidewall, results in:

$$\frac{\partial d}{\partial x} = \tan \theta - \frac{u}{g} \frac{\partial u}{\partial x} - \frac{v}{g} \frac{\partial u}{\partial y} \tag{16}$$

At the sidewall, this equation becomes:

$$\frac{\partial d}{\partial x} = \tan \theta - \frac{u}{g} \frac{\partial u}{\partial x} - \frac{v}{g} \frac{\partial u}{\partial y} - \frac{q_1}{2dg\ \delta y} \frac{\partial u}{\partial x}$$

$$- \frac{q_2\ u}{dg\ \delta x \delta y} \tag{16'}$$

where θ is the angle the channel floor makes with the horizontal and g is the acceleration due to gravity.

The y-direction momentum equation is:

$$\frac{\partial d}{\partial y} = - \frac{v}{g} \frac{\partial v}{\partial y} - \frac{u}{g} \frac{\partial v}{\partial x} \tag{17}$$

for everywhere except at the sidewall.

At the sidewall, Eq. 17 becomes:

$$\frac{\partial d}{\partial y} = -\frac{v}{g}\frac{\partial v}{\partial y} - \frac{u}{g}\frac{\partial v}{\partial x} - \frac{q_1}{2dg\ \delta y}\frac{\partial v}{\partial x} - \frac{q_2\ v}{dg\delta x\delta y} \qquad (17')$$

By substituting Eqs. 16 and 17 into Eq. 15 and simplifying it, gives:

$$(d - \frac{u^2}{g})\frac{\partial u}{\partial x} - \frac{uv}{g}\frac{\partial u}{\partial y} - \frac{uv}{g}\frac{\partial v}{\partial x} + (d - \frac{v^2}{g})\frac{\partial v}{\partial y} = -u \tan\theta \qquad (18)$$

For conditions at the sidewall, substitute Eqs. 16' and 17' into Eq. 15' to obtain:

$$\left(d - \frac{u^2}{g} - \frac{q_1 u}{2dg\ \delta y}\right)\frac{\partial u}{\partial x} + \left(-\frac{uv}{g}\right)\frac{\partial u}{\partial y}$$

$$+ \left(-\frac{uv}{g} - \frac{q_1 v}{2dg\ \delta y}\right)\frac{\partial v}{\partial x} + \left(d - \frac{v^2}{g}\right)\frac{\partial v}{\partial y} = \Phi \qquad (18')$$

where

$$\Phi = \frac{q_1 + q_2}{\delta x\ \delta y} - u \tan\theta + (u^2 + v^2)\frac{1}{dg}\frac{q_2}{\delta x\ \delta y} \qquad (19)$$

The condition for irrotational flow is:

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0 \qquad (20)$$

and the total differentials of u and v are:

$$\delta u = \frac{\partial u}{\partial x}\delta x + \frac{\partial u}{\partial y}\delta y \qquad (21)$$

$$\delta v = \frac{\partial v}{\partial x}\,\delta x + \frac{\partial v}{\partial y}\,\delta y \qquad (22)$$

Eqs. 18, 20, 21, and 22 may be written in matrix form as:

$$\begin{pmatrix} d - \frac{u^2}{g} & -\frac{uv}{g} & -\frac{uv}{g} & d - \frac{v^2}{g} \\ 0 & 1 & -1 & 0 \\ \delta x & \delta y & 0 & 0 \\ 0 & 0 & \delta x & \delta y \end{pmatrix} \begin{pmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \end{pmatrix} = \begin{pmatrix} -u \tan\theta \\ 0 \\ \delta u \\ \delta v \end{pmatrix}$$

or

$$(N) \begin{pmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \end{pmatrix} = \begin{pmatrix} -u \tan\theta \\ 0 \\ \delta u \\ \delta v \end{pmatrix}$$

Applying Cramer's rule, the derivatives of u and v with respect to x and y can be expressed as:

$$\frac{\partial u}{\partial x} = \frac{|K_1|}{|N|}, \quad \frac{\partial u}{\partial y} = \frac{|K_2|}{|N|}, \quad \frac{\partial v}{\partial x} = \frac{|K_3|}{|N|}, \quad \frac{\partial v}{\partial y} = \frac{|K_4|}{|N|} \tag{23}$$

In supercritical transitional flow, discontinuity occurs as a result of the presence of standing waves. Mathematically, such a discontinuity can be described by one of the following conditions:

$$\frac{\partial u}{\partial x} = \frac{0}{0}, \quad \frac{\partial u}{\partial y} = \frac{0}{0}, \quad \frac{\partial v}{\partial x} = \frac{0}{0}, \quad \frac{\partial v}{\partial y} = \frac{0}{0} \tag{24}$$

Equating the determinant of the N matrix with zero results in:

$$\frac{\delta y}{\delta x} = \frac{-uv \pm dg\sqrt{F^2 - 1}}{dg - u^2} \tag{25}$$

where F is the Froude number and equals

$$\frac{V}{\sqrt{gd}} \quad \text{and} \quad V = \sqrt{u^2 + v^2}$$

Equation 25 can be rewritten as

$$\frac{\delta y}{\delta x} = \frac{-uv + dg\sqrt{F^2 - 1}}{dg - u^2} \tag{26a}$$

which is the -c characteristic and

$$\frac{\delta y}{\delta x} = \frac{-uv - dg\sqrt{F^2 - 1}}{dg - u^2} \tag{26b}$$

which is the +c characteristic in the x-y plane.

To define the u-v relationship along the characteristic curves, the determinant of the K_3 matrix is equated to zero.

Solving for $\frac{\delta v}{\delta u}$ gives:

$$\frac{\delta v}{\delta u} = \frac{1}{(d - \frac{v^2}{g})}\left\{-u \tan \theta \ (\frac{\delta y}{\delta u}) - (d - \frac{u^2}{g}) \ (\frac{\delta y}{\delta x})\right\} \qquad (27)$$

replacing $\frac{\delta y}{\delta x}$ with the values from Eqs. 26a and 26b yields:

$$\frac{\delta v}{\delta u} = \frac{1}{(d - \frac{v^2}{g})}\left\{-u \tan \theta \ (\frac{\delta y}{\delta u}) + \frac{uv}{g} - d\sqrt{F^2-1}\right\} \qquad (28a)$$

along the -c characteristic and

$$\frac{\delta v}{\delta u} = \frac{1}{(d - \frac{v^2}{g})}\left\{-u \tan \theta \ (\frac{\delta y}{\delta u}) + \frac{uv}{g} + d\sqrt{F^2-1}\right\} \qquad (28b)$$

along the +c characteristic curves.

For conditions at the sidewall, the x-y relationships become:

$$\frac{\delta y}{\delta x} = \left\{-uv + dg\sqrt{F^2 - 1 + \frac{q_1^2 v^2}{16 d^4 g^2 \delta y^2} + \frac{q_1 u}{2d^2 g \delta y}} + \frac{q_1 v}{4d \delta y}\right\} \div \left\{dg - u^2 - \frac{q_1 u}{2d \delta y}\right\} \qquad (26a')$$

for the -c characteristic and

$$\frac{\delta y}{\delta x} = \left\{-uv - dg\sqrt{F^2-1 + \frac{q_1^2 v^2}{16d^4 g^2 \delta y^2} + \frac{q_1 u}{2d^2 g \delta y}}\right.$$

$$\left. + \frac{q_1 v}{4d \delta y}\right\} \div \left\{dg - u^2 - \frac{q_1 u}{2d \delta y}\right\} \qquad (26b')$$

for the +c characteristic.

The compatibility relationships, as described in Eqs. 28a and 28b become:

$$\frac{\delta v}{\delta u} = \frac{1}{d - \frac{v^2}{g}} \left\{\Phi \frac{\delta y}{\delta u} - \left(d - \frac{u^2}{g} - \frac{q_1 u}{2dg \delta y}\right) \frac{\delta y}{\delta x}\right\} \qquad (28')$$

COMPUTATIONAL PROCEDURES

Since there are four regions, each of which contains different flow conditions, it is necessary to describe the numerical computation procedures in four individual sections. Subdividing the first '-c' characteristic curve into (NP-1) equal y-direction increments, the entire flow field can be isolated into the following computation groups as depicted in Fig. 3. Because of the symmetric flow properties, only half of the test section is shown in Fig. 3.

(I) Region one contains points 0 through NP. Along this characteristic curve $v \equiv 0$, and at point 0 $u = u_o$ and $d = d_o$.

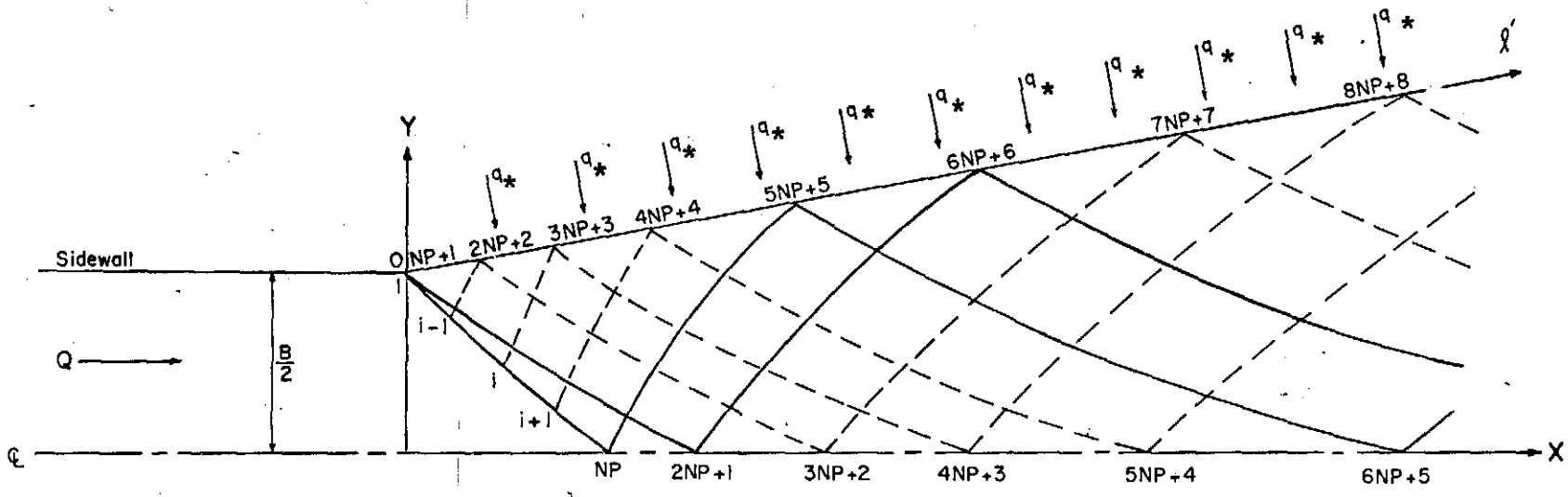


Fig. 3 – Wave Pattern and Characteristic Curves for Supercritical Flow

Based on these conditions, Eq. 28a may be written as:

$$\frac{\delta v}{\delta u} = \frac{1}{d_{i-1}} \left\{ -u_{i-1} \tan \theta \left(\frac{\delta y}{\delta u}\right) - d_{i-1}\sqrt{F_{i-1}^2 - 1} \right\} = 0 \qquad (29)$$

or

$$\delta u = \frac{-u_{i-1} \tan \theta}{d_{i-1}\sqrt{F_{i-1}^2 - 1}} \delta y \qquad (30)$$

Using Eq. 30 with a given δy, δu can be computed such that

$$u_i = u_{i-1} + \delta u \qquad (31)$$

The momentum equation along the negative characteristic line can be written as

$$\rho (V'_{i-1})^2 d_{i-1} \cos \theta + \frac{1}{2} \rho g d_{i-1}^2 \cos \theta + \frac{1}{2} \rho g (d_{i-1} + d_i) \sqrt{(\delta x)^2 + (\delta y)^2} \sin \theta \cos \gamma - \rho (V'_i)^2 d_i \cos \theta - \frac{1}{2} \rho g d_i^2 \cos \theta = 0 \qquad (32)$$

where

$$\gamma = \tan^{-1} \left(\frac{\delta y}{\delta x}\right) \qquad (33)$$

$$V'_{i-1} = \sqrt{(u_{i-1})^2 + (v_{i-1})^2} \cos \left(\gamma - \tan^{-1} \left(\frac{v_{i-1}}{u_{i-1}}\right)\right) \qquad (34)$$

and

$$V_i' = \sqrt{(u_i)^2 + (v_i)^2} \cos\left(\gamma - \tan^{-1}\left(\frac{v_i}{u_i}\right)\right) \tag{34'}$$

Solving for d_i from Eq. 32 results in:

$$d_i = \frac{1}{2}\Bigg\{ - \left[\frac{2}{g}(V_i')^2 - \tan\theta \cos\gamma\sqrt{(\delta x)^2 + (\delta y)^2}\right] + \Bigg\{\left[\frac{2}{g}(V_i')^2 - \tan\theta\cos\gamma\sqrt{(\delta x)^2 + (\delta y)^2}\right]^2 + 4\, d_{i-1}\left[\frac{2}{g}(V_{i-1}')^2 + \tan\theta\cos\gamma\sqrt{(\delta x)^2 + (\delta y)^2} + d_{i-1}\right]\Bigg\}^{1/2}\Bigg\} \tag{35}$$

Rearranging Eq. 26a gives:

$$\delta x = \frac{d_i g - u_i^2}{d_i g\sqrt{F_i^2 - 1}}\,\delta y \tag{36}$$

and

$$x_i = x_{i-1} + \delta x \tag{36'}$$

To increase accuracy, successive approximation may be used for this and all other regions of the numerical computations.

(II) Points JNP + J, along the sidewall constitute the second region.

From the geometry of the channel,

$$\delta x = \frac{y_{i-NP} - \frac{B}{2}}{\tan\alpha} - x_{i-NP} + \frac{\delta y}{\tan\alpha} \tag{37}$$

with B representing the width of the channel where the divergent transition initiates.

Using Eq. 26b', the +c characteristic, we have,

$$\frac{\delta y}{\delta x} = \left\{-u_{i-NP}v_{i-NP} - d_{i-NP}g\sqrt{F^2_{i-NP} - 1 + \frac{q_1^2 v^2_{i-NP}}{16 d^4_{i-NP} g^2 \delta y^2} + \frac{q_1 u_{i-NP}}{2 d^2_{i-NP} g \delta y}}\right.$$

$$\left. + \frac{q_1 v_{i-NP}}{4 d_{i-NP} \delta y}\right\} \div \left\{d_{i-NP}\, g - u^2_{i-NP} - \frac{q_1 u_{i-NP}}{2 d_{i-NP} \delta y}\right\} \qquad (38)$$

Substituting Eq. 37 into Eq. 38 gives:

$$\delta y = \left\{-u_{i-NP} v_{i-NP} \left(\frac{y_{i-NP} - \frac{B}{2}}{\tan \alpha} - x_{i-NP} + \frac{\delta y}{\tan \alpha}\right)\right.$$

$$- g d_{i-NP} \left(\frac{y_{i-NP} - \frac{B}{2}}{\tan \alpha} - x_{i-NP} + \frac{\delta y}{\tan \alpha}\right)$$

$$x \left(\sqrt{F^2_{i-NP} - 1 + \frac{q_1^2 v^2_{i-NP}}{16 d^4_{i-NP} g^2 \delta y^2} + \frac{q_1 u_{i-NP}}{2 d^2_{i-NP} g \delta y}}\right)$$

$$\left. + \frac{q_1 v_{i-NP}}{4 d_{i-NP} \delta y} \left(\frac{y_{i-NP} - \frac{B}{2}}{\tan \alpha} - x_{i-NP} + \frac{\delta y}{\tan \alpha}\right)\right\} \div$$

$$\left\{d_{i-NP} g - u^2_{i-NP} - \frac{q_1 u_{i-NP}}{2 d_{i-NP} \delta y}\right\} \qquad (39)$$

which is solved using interval halving trial and error for the value of δy. Then

$$y_i = y_{i-NP} + \delta y \qquad (40)$$

and

$$x_i = x_{i-NP} + \delta x \tag{40'}$$

Along the sidewall, it is also known that:

$$v_i = u_i \tan \alpha \tag{41}$$

therefore

$$(v_{i-NP} + \delta v) = (u_{i-NP} + \delta u) \tan \alpha \tag{41'}$$

From Eq. 28' we have

$$\frac{\delta v}{\delta u} = \frac{1}{d_{i-NP} - \frac{v_{i-NP}^2}{g}} \left\{ \Phi \frac{y_i - y_{i-NP}}{\delta u} - \left\{ d_{i-NP} - \frac{u_{i-NP}^2}{g} - \frac{q_1 u_{i-NP}}{2 d_{i-NP} g (y_i - y_{i-NP})} \right\} \right.$$

$$\left. \left\{ \frac{y_i - y_{i-NP}}{x_i - x_{i-NP}} \right\} \right\} \tag{42}$$

where

$$\Phi = \frac{q_1 + q_2}{(x_i - x_{i-NP})\ (y_i - y_{i-NP})} + \frac{q_2}{d_{i-NP} g\ (x_i - x_{i-NP})\ (y_i - y_{i-NP})}$$

$$\left\{ u_{i-NP}^2 + v_{i-NP}^2 \right\} - u_{i-NP} \tan \theta \tag{43}$$

Equation 41' gives

$$\delta v = (u_{i-NP} + \delta u) \tan \alpha - v_{i-NP} \tag{41''}$$

Substituting this into Eq. 42 and solving for u_i yields:

$$u_i = \left\{ \Phi(y_i - y_{i-NP}) + v_{i-NP} \left(d_{i-NP} - \frac{v_{i-NP}^2}{g}\right) + u_{i-NP} \left(d_{i-NP} - \frac{u_{i-NP}^2}{g} - \frac{q_1 \, u_{i-NP}}{2 d_{i-NP} g \, (y_i - y_{i-NP})}\right) \frac{y_i - y_{i-NP}}{x_i - x_{i-NP}} \right\} \div$$

$$\left\{ \frac{y_i - y_{i-NP}}{x_i - x_{i-NP}} \left(d_{i-NP} - \frac{u_{i-NP}^2}{g} - \frac{q_1 \, u_{i-NP}}{2 d_{i-NP} g \, (y_i - y_{i-NP})}\right) + \left(d_{i-NP} - \frac{v_{i-NP}^2}{g}\right) \tan \alpha \right\} \qquad (44)$$

The momentum equation along the positive characteristic line can be written as

$$\rho (v'_{i-NP})^2 \, d_{i-NP} \cos \theta + \frac{1}{2} \rho g d_{i-NP}^2 \cos \theta + \frac{1}{2} \rho g \, (d_{i-NP} + d_i) \sqrt{(\delta x)^2 + (\delta y)^2} \sin \theta \cos \gamma - \rho (v'_i)^2 \, d_i \cos \theta$$

$$- \frac{1}{2} \rho g \, d_i^2 \cos \theta = 0 \qquad (45)$$

where

$$v'_{i-NP} = \sqrt{(u_{i-NP})^2 + (v_{i-NP})^2} \cos \left(\gamma - \tan^{-1} \left(\frac{v_{i-NP}}{u_{i-NP}}\right)\right) \qquad (46)$$

Solving Eq. 45 for d_i results in:

$$d_i = \frac{1}{2}\left\{-\left[\frac{2}{g}V_i'^2 - \tan\theta\cos\gamma\sqrt{\delta x^2 + \delta y^2}\right]\right.$$

$$+\left\{\left[\frac{2}{g}V_i'^2 - \tan\theta\cos\gamma\sqrt{\delta x^2 + \delta y^2}\right]^2 + \right.$$

$$\left.\left. 4\, d_{i-NP}\left[\frac{2}{g}V_{i-NP}'^2 + \tan\theta\cos\gamma\sqrt{\delta x^2 + \delta y^2} + d_{i-NP}\right]\right\}^{1/2}\right\} \quad (47)$$

(III) The third region contains those points along the centerline of the channel where $y_i = 0$. Because the flow is symmetrical, $v_i = 0$, everywhere along this line. From Eq. 26a:

$$\delta x = \frac{dg - u^2}{-uv + dg\sqrt{F^2-1}}\ \delta y \quad (48)$$

or

$$\delta x = -y_{i-1}\left(\frac{d_{i-1}\, g - u_{i-1}^2}{-u_{i-1}\, v_{i-1} + d_{i-1}\, g\sqrt{F_{i-1}^2 - 1}}\right) \quad (48')$$

then

$$x_i = x_{i-1} + \delta x \quad (49)$$

Since, along the centerline, $v_i = 0$, $\delta v = -v_{i-1}$.

Solving Eq. 28a for δu yields:

$$\delta u = \frac{-v_{i-1}\left(d - \frac{v^2}{g}\right) - y_{i-1}\, u \tan\theta}{\frac{uv}{g} - d\sqrt{F^2-1}} \qquad (50)$$

The depth along the centerline can now be found by using Eq. 35.

(IV) Region four contains all of the remaining points in the flow field.

For the -c characteristic $\delta y = y_i - y_{i-1}$ and $\delta x = x_i - x_{i-1}$. For the +c characteristic $\delta y = y_i - y_{i-NP}$ and $\delta x = x_i - x_{i-NP}$.

Substituting these values into Eqs. 26a and 26b gives:

$$\frac{y_i - y_{i-1}}{x_i - x_{i-1}} = \frac{-u_{i-1}\, v_{i-1} + d_{i-1}\, g\sqrt{F_{i-1}^2 - 1}}{d_{i-1} g - u_{i-1}^2} = G^- \qquad (52)$$

and

$$\frac{y_i - y_{i-NP}}{x_i - x_{i-NP}} = \frac{-u_{i-NP} v_{i-NP} - d_{i-NP}\, g\sqrt{F_{i-NP}^2 - 1}}{d_{i-NP} g - u_{i-NP}^2} = G^+ \qquad (53)$$

from Eq. 52

$$y_i = G^- (x_i - x_{i-1}) + y_{i-1} \qquad (54)$$

Substituting Eq. 54 into Eq. 53 and rearranging yields:

$$x_i = \frac{G^- x_{i-1} - G^+ x_{i-NP} - y_{i-1} + y_{i-NP}}{G^- - G^+} \qquad (55)$$

By substituting this value of x_i into Eq. 54, y_i can be determined.

Along the -c characteristic, $\delta u)_n = u_i - u_{i-1}$ and $\delta v)_n = v_i - v_{i-1}$ and along the +c characteristic $\delta u)_p = u_i - u_{i-NP}$ and $\delta u)_p = v_i - v_{i-NP}$.

therefore

$$\delta u)_n - \delta u)_p = u_{i-NP} - u_{i-1} = G1 \tag{56}$$

and

$$\delta v)_n - \delta v)_p = v_{i-NP} - v_{i-1} = G2 \tag{56'}$$

Let

$$\frac{-u_{i-1} \tan \theta \ (y_i - Y_{i-1})}{d_{i-1} - \frac{v_{i-1}^2}{g}} = G3 \tag{57}$$

$$\frac{-u_{i-NP} \tan \theta \ (y_i - y_{i-NP})}{d_{i-NP} - \frac{v_{i-NP}^2}{g}} = G5 \tag{57'}$$

$$\frac{1}{d_{i-1} - \frac{v_{i-1}^2}{g}} \left\{ \frac{u_{i-1} \ v_{i-1}}{g} - d_{i-1} \sqrt{F_{i-1}^2 - 1} \right\} = G4 \tag{58}$$

and

$$\frac{1}{d_{i-NP} - \frac{v_{i-NP}^2}{g}} \left\{ \frac{u_{i-NP}\, v_{i-NP}}{g} + d_{i-NP}\sqrt{F_{i-NP}^2 - 1} \right\} = G6 \quad (58')$$

Eqs. 11a' and 11b' become:

$$\frac{\delta v)_n}{\delta u)_n} = \frac{G\ 3}{\delta u)_n} + G4 \quad (59)$$

and

$$\frac{\delta v)_p}{\delta u)_p} = \frac{G\ 5}{\delta u)_p} + G6 \quad (60)$$

Combining Eqs. 56. 56', 59, and 60 yields:

$$\delta u)_p = \frac{G5 + G2 - G3 - (G1)\ (G4)}{G4 - G6} \quad (61)$$

then

$$u_i = u_{i-NP} + \delta u)_p \quad (62)$$

From Eq. 60 $\delta v)_p$ can be found.

Then

$$v_i = v_{i-NP} + \delta v)_p \quad (63)$$

An equation similar to Eq. 32 can be written along the +c characteristic, resulting in

$$d_i = \frac{1}{2}\left\{ - \left[\frac{2}{g}(V_i')^2 - \tan\theta \cos\gamma\sqrt{(\delta x)^2 + (\delta y)^2}\right] + \left\{\left[\frac{2}{g}(V_i')^2 - \tan\theta\cos\gamma\sqrt{(\delta x)^2 + (\delta y)^2}\right]^2 + 4\, d_{i-NP}\left[\frac{2}{g}(V'_{i-NP})^2 + \tan\theta\cos\gamma\sqrt{(\delta x)^2 + (\delta y)^2} + d_{i-NP}\right]\right\}^{1/2}\right\} \quad (64)$$

DETERMINATION OF BOUNDARY SHEAR IN OPEN CHANNEL FLUMES

The expression for the boundary shear stress, τ, was determined by combining several other equations which relate frictional resistance to the flow characteristics.

From the Chezy equation (8) we have:

$$\bar{V} = C\sqrt{R\,S} \quad (65)$$

where $\bar{V}$ is the mean velocity, C is the friction factor, R is the hydraulic radius and S is the slope of the energy grade line.

Modern fluid dynamics and especially boundary layer theory (15) suggest Chezy's formula to be used in the form

$$\bar{V} = \sqrt{\frac{8}{f}}\ \sqrt{g\,R\,S} \quad (66)$$

where f is the Darcy-Weisbach friction factor, and g is the acceleration due to gravity.

Setting Eq. 65 equal to Eq. 66 yields:

$$C = \sqrt{\frac{8g}{f}} \tag{67}$$

The relationship between Manning's roughness factor, n, and Chezy's C (9) is known as:

$$C = \frac{1.49}{n} R^{1/6} \tag{68}$$

By equating Eqs. 67 and 68 we have

$$f = \frac{3.603\ gn^2}{R^{1/3}} \tag{69}$$

The relationship (16) between shear stress, friction factor and mean velocity is:

$$\sqrt{\frac{\tau}{\rho}} = \sqrt{\frac{f}{8}}\ \bar{V} \tag{70}$$

in which ρ is the mass density of the fluid.

Substituting Eq. 69 into Eq. 70 and solving for τ yields:

$$\tau = \frac{0.45\ \gamma\ n^2\ V^2}{R^{1/3}} \tag{71}$$

in which γ is the specific weight of the fluid.

The above expression is used to determine the average boundary shear stress, τ, when the Manning roughness coefficient, n, is precalibrated for the flume used in the experimental work.

CHAPTER III

EXPERIMENTAL FACILITY

The experimental facility used in this study consists of five major parts: main flow channel, convergent-divergent test section, plexiglas overflow devices for providing uniform lateral inflow to the divergent test section, flow supply systems, and measuring devices.

A seventy-two feet long glass flume with a two feet by two feet cross section is used as the main flow channel. This flume consists of three separate sections. Four pairs of electric motor operated gear jacks are used to support and control the slope of each section. One pair of gear jacks are used for each end section and two for the middle section. A sluice gate at the beginning of the flume is used to regulate the inflow to the main channel. The constant flowrate is supplied by a constant head storage tank behind the sluice gate. A schematic drawing of the flume and experimental setup is shown in Fig. 4.

To gradually reduce the width of the main flow channel from two feet to one foot, two pieces of sheet metal are used to form the convergent section. The divergent test section is formed by using two boards,

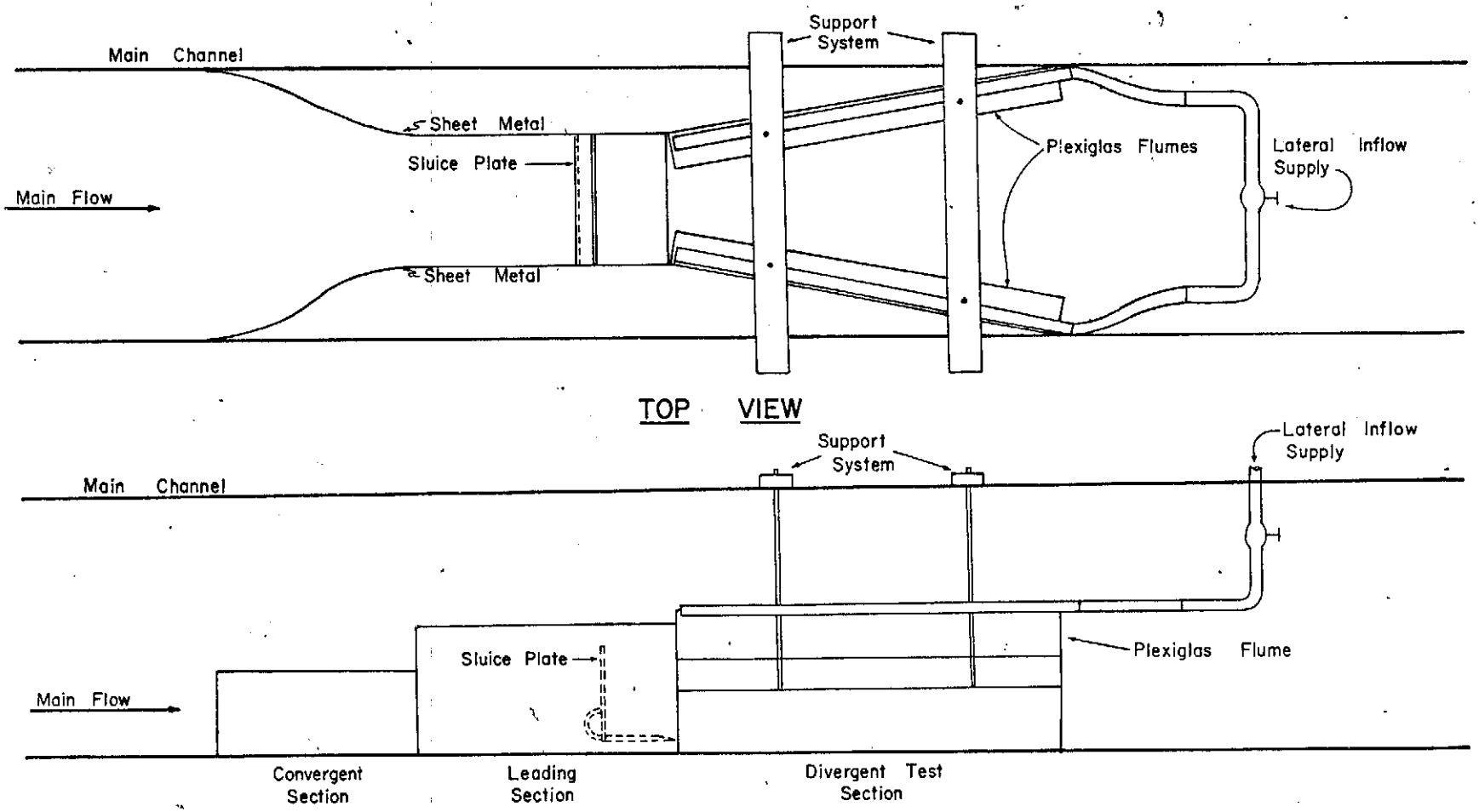


Fig. 4 – Main Channel with Experimental Converging-Diverging Apparatus

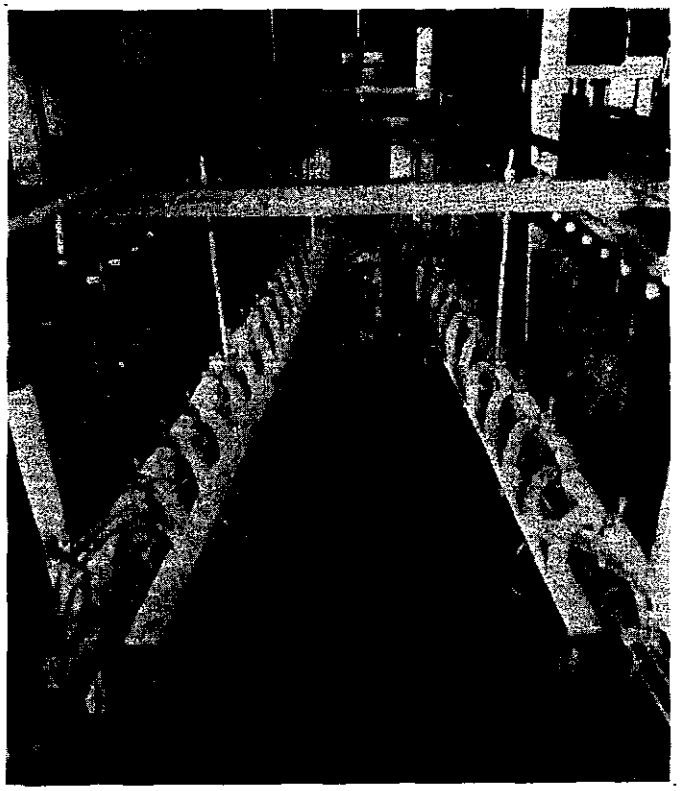


Fig. 5 Experimental Apparatus

six feet long, leading from the constricted width to the normal flume width of two feet. Between the convergent section and the test section, a four feet parallel wall section is inserted to form uniform approaching flow to the test section. In order to obtain nearly uniform velocity distribution at the entrance of the test section, a sluice plate is used at various locations until the velocity measurements indicate the velocity throughout the entrance section varies no more than one percent from its mean value. Figure 4 shows the general arrangement of the convergent-divergent section and the lateral flow supply devices.

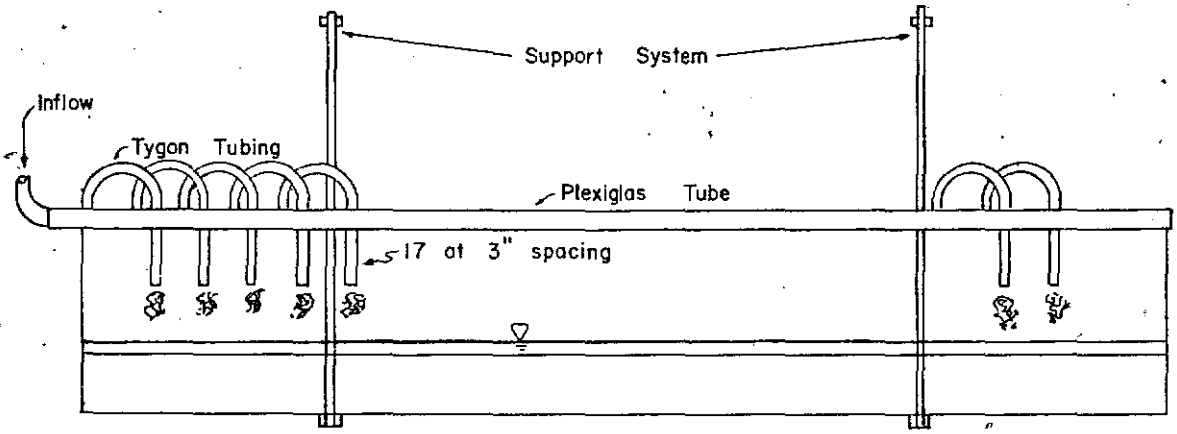
The lateral flow supply devices are plexiglas flumes, constructed so as to provide uniform sheet flow along the side walls of the divergent test section. A knife edged overflow weir along the entire front of the flume permits water to flow over and form a water sheet on an inclined plate set at fifteen degrees from the vertical. The lower edge of the inclined plate is placed at a uniform gap from the side wall to allow the sheet flow to enter the test section. A one-eighth inch plexiglas partition was inserted three-quarters of an inch from the back of the flume to separate the flume into two compartments. This partition was placed one-eighth inch from the bottom to produce a head in the back compartment

and to reduce turbulence in the front compartment. To further reduce the turbulence, a baffle strip was placed in the front compartment along the front wall. Figure 6 shows the plexiglas flume and its cross section.

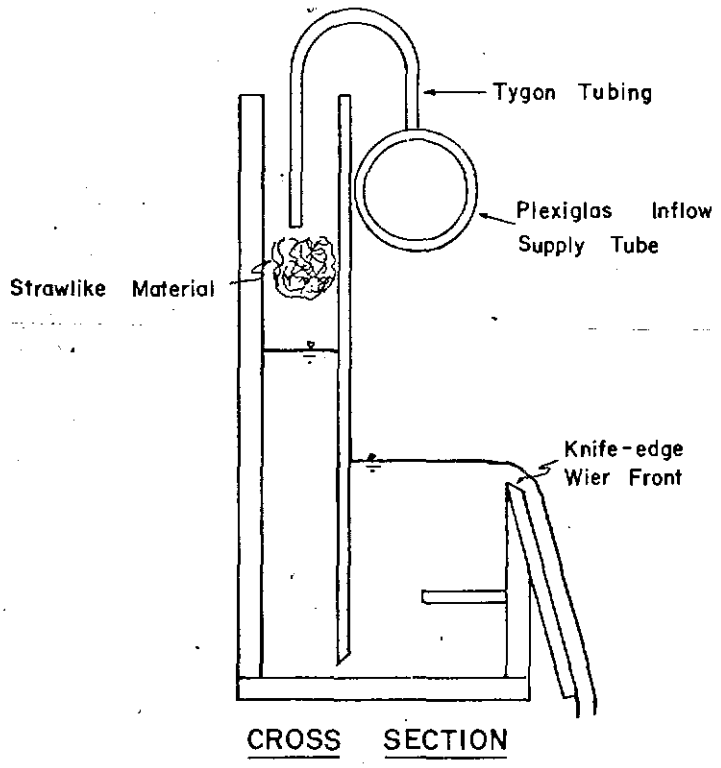
Each plexiglas flume is suspended from the main channel side walls by two, one-half inch stainless steel rods, threaded at each end. This allows adjustment of the flumes up and down to maintain their level position while the main channel slope is varied.

The lateral flow discharge is supplied through one inch galvanized pipe to the plexiglas flumes. A valve in the system regulates the flow to the flumes. To measure lateral flowrate, a pre-calibrated elbow meter was installed in the line. A mercury manometer is used to measure the differential pressure in the elbow meter. The galvanized pipe is then branched to two plexiglas tubes connected with tygon tubing. In the top of each plexiglas tube are distribution holes set at approximately three inches apart. Tygon tubing from each of these holes is used to carry the water into the back compartment of each plexiglas flume. Strawlike material was placed in the back compartment under each jet to dissipate the turbulence caused by the water jets.

An electronic point gage was used to measure the water depth. The point gage is mounted on a carriage



FRONT VIEW



CROSS SECTION

Fig. 6 - Uniform Lateral Inflow Plexiglas Flume

which slides on two pieces of tubular conduit. The conduit is mounted on each side of the main channel. A scale on one conduit determines the longitudinal location whereas a scale on the carriage determines the lateral location of the point gage. The point gage is equipped with a vernier scale to produce readings of $\pm$ 5 x 10^{-4} feet accuracy.

CHAPTER IV

EXPERIMENTAL PROCEDURE

For the experimental data collection, the constant head storage tank and its associated valves were adjusted such that a given flowrate is provided in the main channel. For each main channel flowrate, a maximum lateral inflow of 0.0242 cfs/ft. was first supplied to the diverging zone. This maximum value provides a total lateral inflow over the test divergent section of approximately one half the magnitude of the main channel flow. The lateral inflow was then reduced to 50% of the maximum for a second set of data gathering. The third data set was taken for the water surface profile of the same flowrate without lateral inflow. The main channel flowrate was then readjusted. The above procedure was repeated using the new flow. A slope of 1.5% was used for the various trials in subcritical flow region.

A uniform transverse water surface was assumed for subcritical flow conditions. Therefore, only centerline measurements were taken when the flow is subcritical.

A similar procedure was followed for supercritical flow measurements. However, the presence of standing waves

in supercritical flow makes the assumption of a constant transverse water surface invalid. Therefore, a two dimensional coordinate system, as shown in Figure 3, was selected to record these wave patterns on an x - y plane. Detailed centerline measurements were also recorded.

To study the effect of slope, two different slopes were tried. With a minimum slope of 1.0%, the described procedure was followed; then repeated, using a slope of 4.0%.

In order to measure the water depth, elevation of the channel floor was first determined. This was done by the use of an electronic point gage, which can be moved freely in the x-y plane, set along the top of the main channel. This elevation was then subtracted from the water surface elevation measurement to give the net water depth at each desired point.

Flowrates for both the main channel and the plexiglas flumes were determined using mercury-water manometers. For the main channel, the manometer is attached to a Venturi-Meter and for the lateral inflow supply flumes, it is connected to an elbow meter.

CHAPTER V

DATA ANALYSIS AND PRESENTATION

The direct measurements taken during the course of this study include the channel floor elevation; the water surface elevation along the centerline of the divergent test section; the head difference on the manometers attached to the Venturi and elbow meters, Δh_Q and Δh_{q_*}; and the downstream distance of each measurement from the entrance to the divergent test section. The precalibrated flowrate curves for the Venturi and elbow meters were used to determine the main channel flowrate, Q, and the lateral inflow rate, q_*. The water depth was determined by substracting the channel floor elevation from the water surface elevation.

ANALYSIS OF SUBCRITICAL SPATIALLY VARIED FLOW

For the case of subcritical spatially varied flow, a constant transverse water surface was assumed. Thus, only the analytical and experimental centerline profiles are compared. With the aid of a digital computer, the analytical water surface profile is determined for each group of initial conditions. The program used in the computations was written using Fortran IV language and is listed in Appendix III. A flow chart corresponding to this program is shown

in Fig. A-1. For each data set there are two corresponding plots. These plots were produced by the Calcomp Plotter with OS/360.

The analytical profile was computed from Eq. 14. The initial depth used in the analytical computations is the initial experimental depth and the incremental downstream distance used is 0.1 feet. Six sets of data were taken and the results are presented in Tables I and II, and graphically shown in Figures 7-10.

Table I shows the results for the first three data sets. For these data sets, a main channel flowrate of 0.368 cfs was used. Other parameters held constant were the channel slope, 1.5%; the divergent angle, 4.17°; and the total downstream distance, 6 feet.

With these parameters constant, the three sets of computations shown in each Table are (A) $q_* = 0.0$ cfs/ft, (B) $q_* = 0.0125$ cfs/ft, and (C) $q_* = 0.0242$ cfs/ft. Listed in each group are the analytical and experimental depths at each 0.5 foot interval. Also listed is the percent difference between the analytical and experimental depth.

The corresponding plots for these data sets are shown in Figs. 7 and 8. Figure 7 shows the relationship between water depth and downstream distance. This and all subsequent plots are plotted on arithmetic scales. The

normalized results, by dividing the water depth by the initial depth and dividing the downstream distance by the initial width, are shown in Fig. 8. For Figs. 7 and 8, (A), (B), and (C) correspond to those results listed in Table I.

Table II lists the results for the second three data sets. For these sets, all initial parameters were kept the same as used in previous sets except the main channel flow-rate which is changed to 0.211 cfs. The same lateral inflow rates as listed in Table I apply to Table II, (A), (B), and (C).

Figure 9 shows the relationship between water depth and downstream distance and the corresponding dimensionless plot is shown in Fig. 10.

Figures 7 through 10 indicate the largest difference between the analytical and experimental centerline profiles occurs near the end of the test section. Tables I and II verify this, showing the largest percent difference occurring at a downstream distance of 6 feet in all cases.

ANALYSIS OF SUPERCRITICAL SPATIALLY VARIED FLOW

The method of characteristics, which is used widely in analyzing supersonic gas flow, is used for the analysis of supercritical spatially varied flow. In applying this method, incremental changes along the characteristic curves are computed so that the flow information at various points along the curves can be obtained. The computed results for

each data set are presented in Tables III through XIV. Listed in each table are the points of interest and their corresponding location in the x-y coordinate system used; the velocities in both the x and y directions; and the water depth at each point.

A digital computer program (see Appendix III), written also in Fortran IV, is used for the analytical computations. Also used is the Calcomp Plotter to display the results in graphical form. The graphical results are shown in Figures 12 through 19. For each data set, there are two plots. They are the analytical and experimental centerline profiles and the analytical cross wave patterns. Experimental values measured at the entrance of the test section are used as the initial conditions in the computational analysis.

Table III shows the results for the first supercritical data set. For this set, the main channel flowrate is 0.3632 cfs, the lateral inflow is zero, and the channel slope is 1%. Holding the main channel flowrate and the slope constant, the lateral inflow is increased to 0.0236 cfs/ft for the results presented in Table V, and then decreased to 0.0125 cfs/ft in Table IV. The corresponding centerline profiles are shown in Figure 12 and the analytical cross wave patterns in Figure 13. Holding all parameters constant except the channel slope, which is changed to 4%, the results in Tables VI, VII and VIII are

produced. The corresponding centerline profiles and cross wave patterns are shown in Figures 14 and 15, respectively.

The same channel slopes and lateral inflow combinations were used for a 0.1967 cfs main channel flowrate for further experimental observations. These results are listed in Tables IX - XIV. The corresponding plots of centerline profiles and cross wave patterns are displayed in Figures 16-19.

The results for the supercritical data sets show that the analytical and experimental centerline profiles follow closely the same general trend. As can be seen from Figures 12, 14, 16, and 18, the largest difference is for the case of a 1% channel slope and the maximum lateral inflow rate.

TABLE I ANALYTICAL RESULTS FOR SUBCRITICAL FLOW

MAIN CHANNEL FLOWRATE: 0.368 CFS

(A) LATERAL INFLOW: 0.0 CFS/FT

DOWNSTREAM DISTANCE (FT)	DEPTH (FT) ANALYTICAL	EXPERIMENTAL	PERCENT DIFFERENCE
0.0	0.2730	0.273	0.0
0.50	0.2861	0.285	0.385
1.00	0.2976	0.296	0.534
1.50	0.3081	0.306	0.668
2.00	0.3178	0.314	1.205
2.50	0.3271	0.324	0.952
3.00	0.3360	0.331	1.499
3.50	0.3447	0.340	1.360
4.00	0.3531	0.349	1.170
4.50	0.3614	0.357	1.220
5.00	0.3696	0.364	1.505
5.50	0.3776	0.371	1.750
6.00	0.3856	0.378	1.964

(B) LATERAL INFLOW: 0.0125 CFS/FT

DOWNSTREAM DISTANCE (FT)	DEPTH (FT) ANALYTICAL	EXPERIMENTAL	PERCENT DIFFERENCE
0.0	0.3060	0.306	0.0
0.50	0.3177	0.317	0.212
1.00	0.3284	0.325	1.028
1.50	0.3384	0.334	1.301
2.00	0.3479	0.340	2.282
2.50	0.3571	0.348	2.551
3.00	0.3660	0.358	2.186
3.50	0.3747	0.366	2.314
4.00	0.3832	0.371	3.174
4.50	0.3915	0.381	2.686
5.00	0.3998	0.388	2.940
5.50	0.4079	0.394	3.407
6.00	0.4160	0.401	3.596

(C) LATERAL INFLOW: 0.0242 CFS/FT

DOWNSTREAM DISTANCE (FT)	DEPTH (FT) ANALYTICAL	EXPERIMENTAL	PERCENT DIFFERENCE
0.0	0.3120	0.312	0.0
0.50	0.3235	0.321	0.779
1.00	0.3342	0.328	1.862
1.50	0.3443	0.335	2.713
2.00	0.3540	0.341	3.676
2.50	0.3634	0.350	3.674
3.00	0.3724	0.358	3.871
3.50	0.3813	0.365	4.268
4.00	0.3900	0.373	4.347
4.50	0.3985	0.381	4.388
5.00	0.4069	0.387	4.891
5.50	0.4152	0.393	5.350
6.00	0.4234	0.400	5.536

TABLE II ANALYTICAL RESULTS FOR SUBCRITICAL FLOW

MAIN CHANNEL FLOWRATE: 0.211 CFS

(A) LATERAL INFLOW: 0.0 CFS/FT

DOWNSTREAM DISTANCE (FT)	DEPTH (FT) ANALYTICAL	EXPERIMENTAL	PERCENT DIFFERENCE
0.0	0.2130	0.213	0.0
0.50	0.2235	0.224	0.220
1.00	0.2332	0.231	0.935
1.50	0.2423	0.238	1.776
2.00	0.2510	0.245	2.408
2.50	0.2595	0.253	2.510
3.00	0.2678	0.261	2.533
3.50	0.2759	0.268	2.864
4.00	0.2839	0.276	2.784
4.50	0.2918	0.282	3.363
5.00	0.2997	0.289	3.556
5.50	0.3074	0.295	4.046
6.00	0.3152	0.301	4.497

(B) LATERAL INFLOW: 0.0125 CFS/FT

DOWNSTREAM DISTANCE (FT)	DEPTH (FT) ANALYTICAL	EXPERIMENTAL	PERCENT DIFFERENCE
0.0	0.2790	0.279	0.0
0.50	0.2881	0.287	0.373
1.00	0.2968	0.295	0.622
1.50	0.3054	0.300	1.767
2.00	0.3138	0.306	2.476
2.50	0.3220	0.314	2.487
3.00	0.3301	0.322	2.465
3.50	0.3382	0.328	3.011
4.00	0.3462	0.335	3.222
4.50	0.3541	0.342	3.407
5.00	0.3619	0.348	3.847
5.50	0.3697	0.354	4.258
6.00	0.3775	0.361	4.378

(C) LATERAL INFLOW: 0.0242 CFS/FT

DOWNSTREAM DISTANCE (FT)	DEPTH (FT) ANALYTICAL	EXPERIMENTAL	PERCENT DIFFERENCE
0.0	0.3260	0.326	0.0
0.50	0.3346	0.334	0.181
1.00	0.3431	0.341	0.602
1.50	0.3514	0.347	1.253
2.00	0.3596	0.354	1.570
2.50	0.3678	0.361	1.850
3.00	0.3759	0.368	2.099
3.50	0.3839	0.375	2.323
4.00	0.3919	0.382	2.526
4.50	0.3998	0.387	3.209
5.00	0.4077	0.393	3.612
5.50	0.4156	0.399	3.991
6.00	0.4234	0.405	4.350

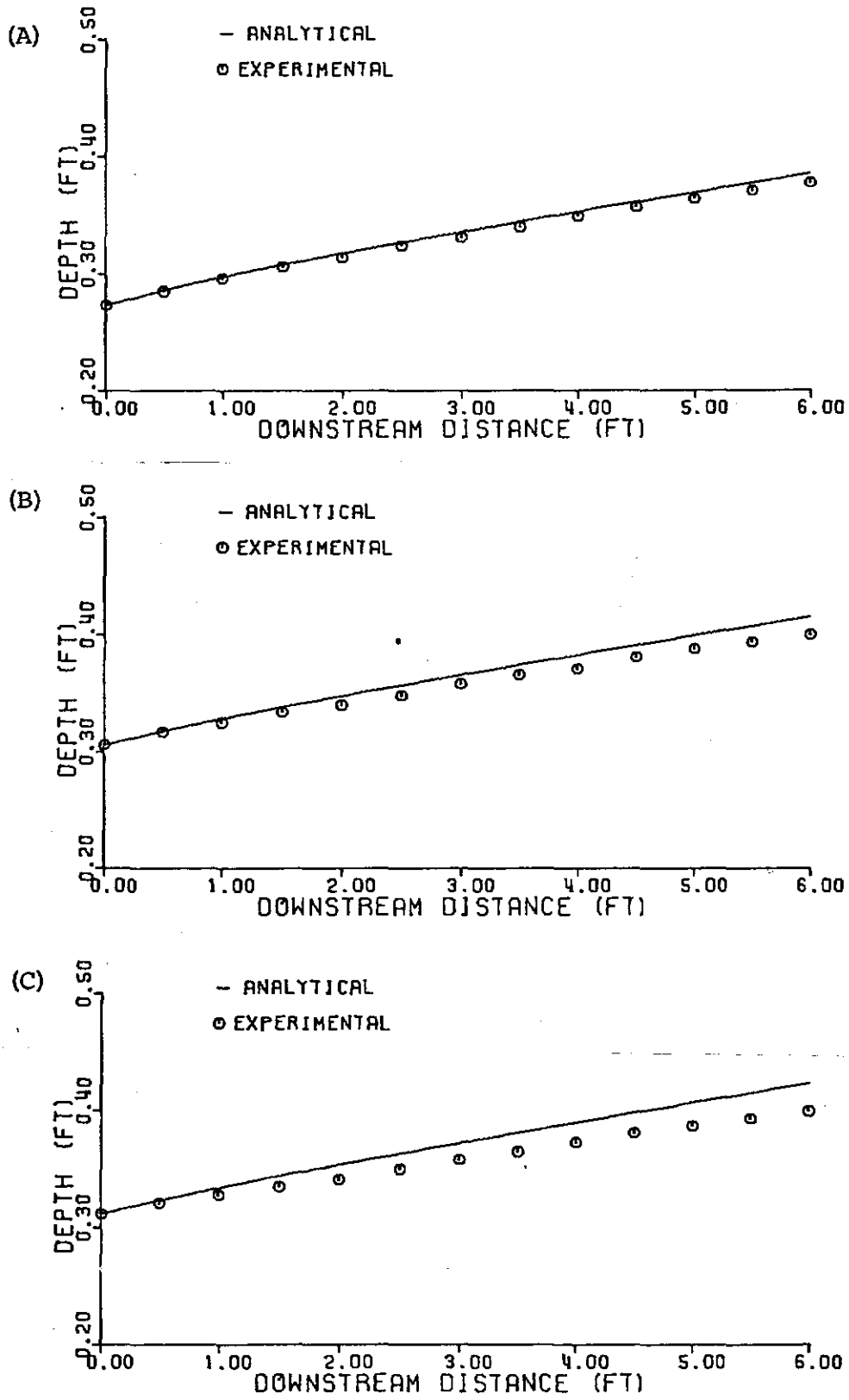


Fig. 7 Relationship Between Depth and Downstream Distance for Subcritical Flow; Q = 0.368 cfs: (A) $q_* = 0.0$ cfs/ft (B) $q_* = 0.0125$ cfs/ft (C) $q_* = 0.0242$ cfs/ft

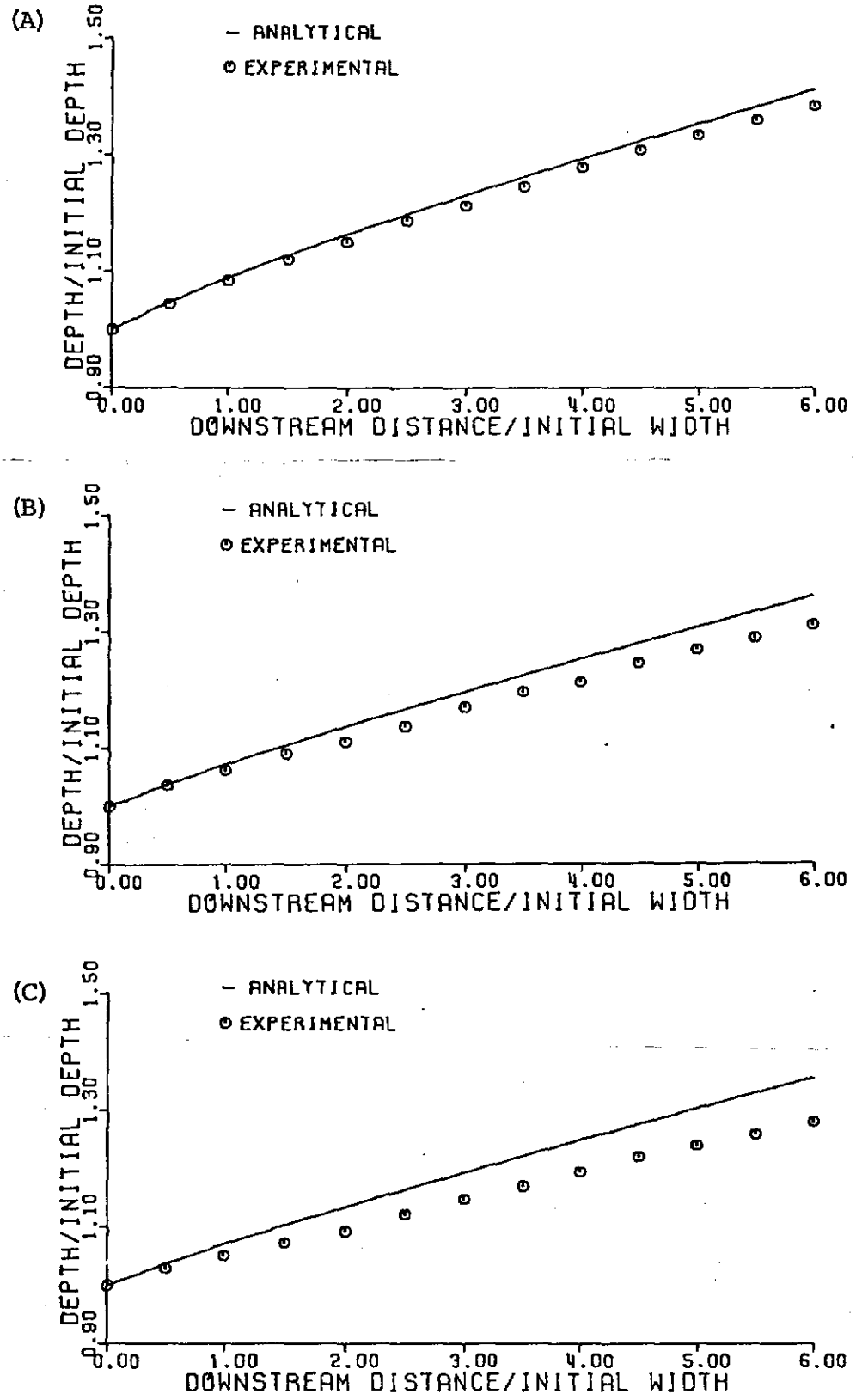


Fig. 8 Dimensionless Relationship Between Depth and Downstream Distance for Subcritical Flow; Q = 0.368 cfs: (A) $q_* = 0.0$ cfs/ft (B) $q_* = 0.0125$ cfs/ft (C) $q_* = 0.0242$ cfs/ft

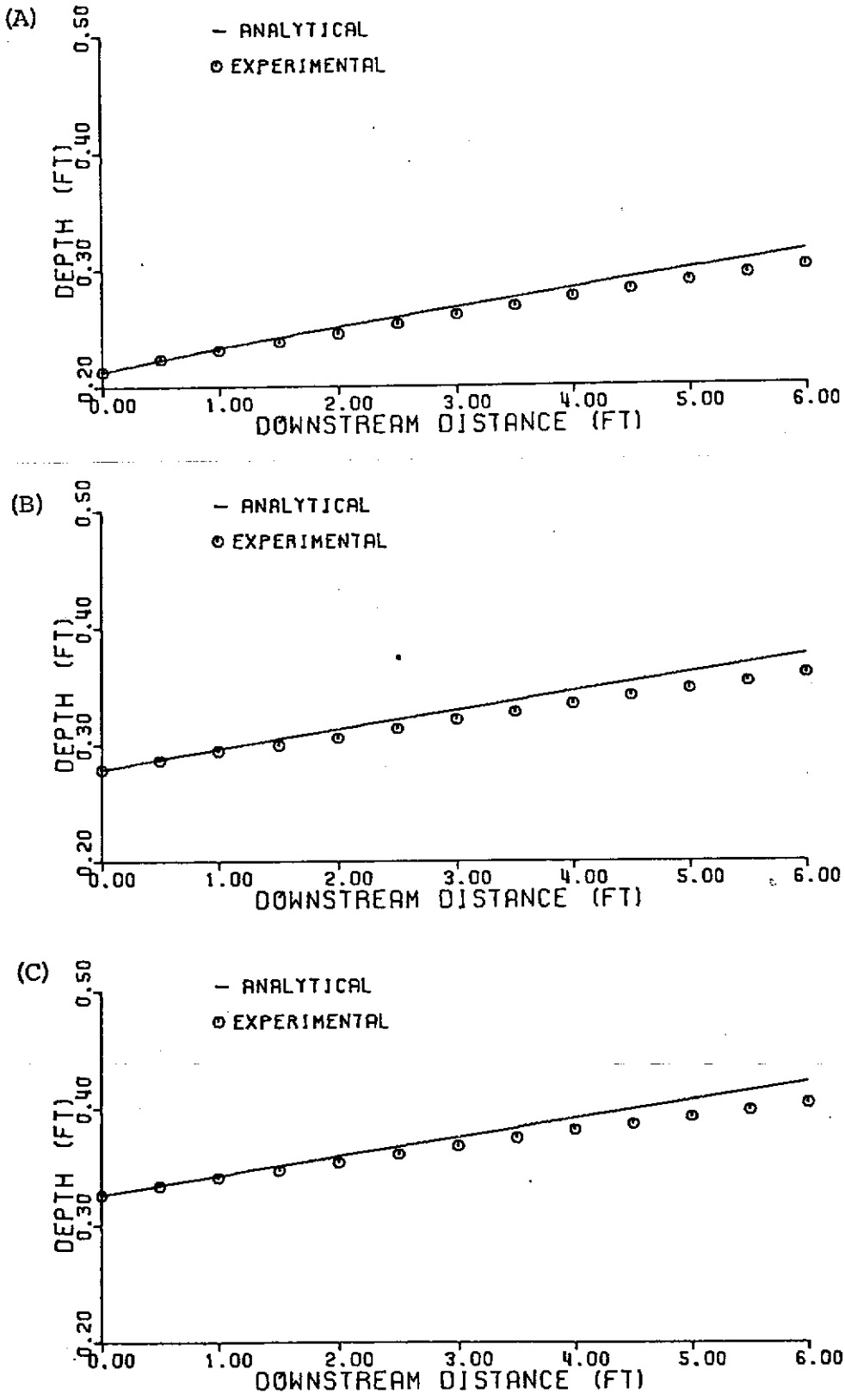


Fig. 9 Relationship Between Depth and Downstream Distance for Subcritical Flow; Q = 0.211 cfs: (A) $q_* = 0.0$ cfs/ft (B) $q_* = 0.0125$ cfs/ft (C) $q_* = 0.0242$ cfs/ft

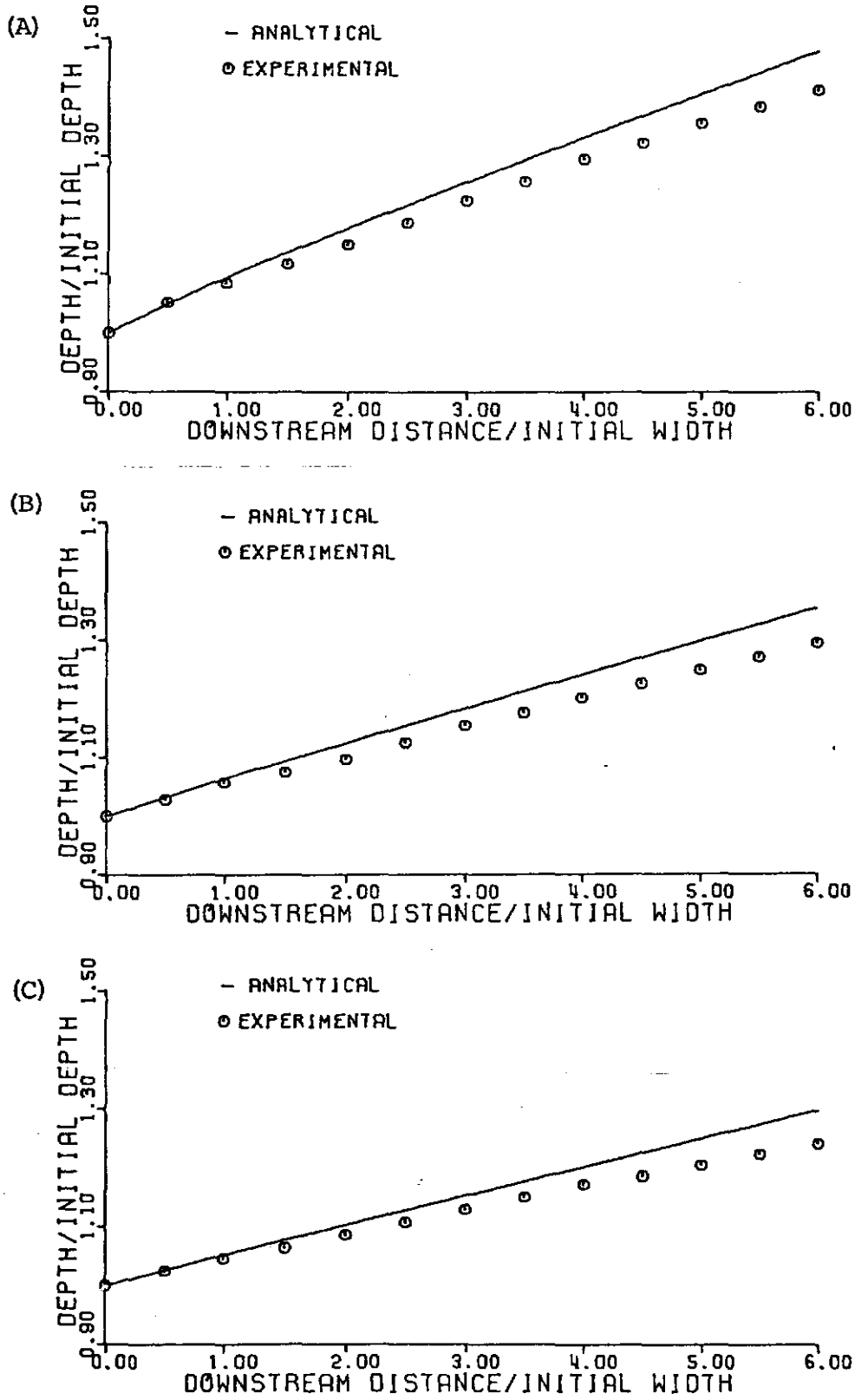


Fig. 10 Dimensionless Relationship Between Depth and Downstream Distance for Subcritical Flow; Q = 0.211 cfs: (A) q_* = 0.0 cfs/ft (B) q_* = 0.0125 cfs/ft (C) q_* = 0.0242 cfs/ft

(A) $q_* = 0.0$ cfs/ft

(B) $q_* = 0.0242$ cfs/ft

Fig 11 Typical Cross Waves of Supercritical Spatially Varied Flow
(A) Without Lateral Inflow (B) With Lateral Inflow

TABLE III ANALYTICAL RESULTS FOR SUPERCRITICAL FLOW

MAIN CHANNEL FLOWRATE: 0.3632 CFS LATERAL INFLOW: 0.0 CFS/FT SLOPE: 1.00 %

POINT	DOWNSTREAM DISTANCE (FT)	TRANSVERSE DISTANCE (FT)	VELOCITY (FPS) X-DIRECTION	VELOCITY (FPS) Y-DIRECTION	DEPTH (FT)
0	0.0	0.500	2.9770	0.0	0.1220
1	0.025	0.478	2.9818	0.0	0.1219
2	0.161	0.359	3.0077	0.0	0.1213
3	0.299	0.239	3.0335	0.0	0.1207
4	0.440	0.120	3.0590	0.0	0.1201
5	0.584	0.000	3.0845	0.0	0.1195
6	0.056	0.505	3.1773	0.2648	0.1036
7	0.221	0.409	3.2037	0.2628	0.1033
8	0.389	0.313	3.2299	0.2610	0.1030
9	0.560	0.217	3.2560	0.2593	0.1027
10	0.734	0.120	3.2821	0.2576	0.1024
11	0.950	0.0	3.4800	0.0	0.0886
12	0.371	0.531	3.2293	0.2691	0.1025
13	0.542	0.435	3.2555	0.2672	0.1023
14	0.716	0.339	3.2816	0.2655	0.1020
15	0.893	0.243	3.3077	0.2638	0.1017
16	1.143	0.107	3.5070	0.0070	0.0879
17	1.341	0.0	3.5342	0.0	0.0871
18	0.697	0.558	3.2810	0.2734	0.1015
19	0.874	0.463	3.3071	0.2716	0.1012
20	1.054	0.367	3.3332	0.2699	0.1009
21	1.340	0.214	3.5340	0.0139	0.0872
22	1.541	0.107	3.5612	0.0069	0.0864
23	1.746	0.0	3.5884	0.0	0.0857
24	1.036	0.586	3.3326	0.2777	0.1005
25	1.219	0.491	3.3586	0.2759	0.1002
26	1.541	0.323	3.5610	0.0207	0.0864
27	1.746	0.216	3.5883	0.0137	0.0857
28	1.955	0.108	3.6156	0.0069	0.0850
29	2.168	0.0	3.6429	0.0	0.0843
30	1.386	0.615	3.3840	0.2820	0.0994
31	1.745	0.432	3.5880	0.0276	0.0857
32	1.954	0.325	3.6154	0.0206	0.0850
33	2.167	0.218	3.6428	0.0137	0.0843
34	2.384	0.109	3.6702	0.0068	0.0836
35	2.605	0.0	3.6976	0.0	0.0829
36	2.208	0.684	3.7612	0.3134	0.0753
37	2.459	0.595	3.7911	0.3059	0.0747
38	2.715	0.505	3.8210	0.2985	0.0742
39	2.975	0.414	3.8510	0.2912	0.0737
40	3.241	0.322	3.8810	0.2840	0.0731
41	4.130	0.0	4.0908	0.0	0.0649
42	2.697	0.725	3.8196	0.3183	0.0740
43	2.958	0.635	3.8496	0.3109	0.0735
44	3.224	0.545	3.8798	0.3036	0.0730
45	3.496	0.453	3.9101	0.2963	0.0724
46	4.444	0.118	4.1235	0.0125	0.0643
47	4.766	0.0	4.1567	0.0	0.0637
48	3.206	0.767	3.8783	0.3232	0.0728
49	3.478	0.677	3.9087	0.3159	0.0723
50	3.755	0.586	3.9391	0.3086	0.0718
51	4.765	0.238	4.1564	0.0250	0.0637
52	5.095	0.120	4.1898	0.0124	0.0630
53	5.431	0.0	4.2233	0.0	0.0624
54	3.736	0.811	3.9376	0.3281	0.0716
55	4.019	0.721	3.9683	0.3208	0.0711
56	5.092	0.360	4.1894	0.0374	0.0630
57	5.430	0.242	4.2232	0.0249	0.0624
58	5.774	0.122	4.2570	0.0125	0.0618
59	6.126	0.0	4.2911	0.0	0.0612
60	4.288	0.857	3.9975	0.3331	0.0705
61	5.426	0.484	4.2226	0.0500	0.0624
62	5.772	0.366	4.2567	0.0374	0.0618
63	6.125	0.246	4.2909	0.0250	0.0612
64	6.485	0.124	4.3253	0.0125	0.0606
65	6.853	0.0	4.3600	0.0	0.0600
66	6.950	1.079	4.4389	0.3699	0.0558

TABLE IV ANALYTICAL RESULTS FOR SUPERCRITICAL FLOW

MAIN CHANNEL FLOWRATE: 0.3632 CFS LATERAL INFLOW: 0.01250 CFS/FT SLOPE: 1.00 %

POINT	DOWNSTREAM DISTANCE (FT)	TRANSVERSE DISTANCE (FT)	VELOCITY (FPS) X-DIRECTION	VELOCITY (FPS) Y-DIRECTION	DEPTH (FT)
0	0.0	0.500	2.9770	0.0	0.1220
1	0.025	0.478	2.9818	0.0	0.1219
2	0.161	0.359	3.0077	0.0	0.1213
3	0.299	0.239	3.0335	0.0	0.1207
4	0.440	0.120	3.0590	0.0	0.1201
5	0.584	0.000	3.0845	0.0	0.1195
6	0.054	0.504	3.0032	0.2503	0.1111
7	0.208	0.399	3.1258	0.1418	0.1108
8	0.364	0.293	3.1515	0.1408	0.1104
9	0.522	0.186	3.1773	0.1398	0.1100
10	0.683	0.080	3.2029	0.1389	0.1096
11	0.805	0.0	3.3195	0.0	0.1007
12	0.359	0.530	3.0448	0.2537	0.1107
13	0.522	0.422	3.1733	0.1359	0.1102
14	0.683	0.315	3.1989	0.1349	0.1098
15	0.846	0.207	3.2244	0.1340	0.1093
16	0.987	0.116	3.3418	-0.0042	0.1004
17	1.170	0.0	3.3638	0.0	0.1001
18	0.684	0.557	3.0981	0.2582	0.1096
19	0.853	0.448	3.2260	0.1368	0.1091
20	1.019	0.341	3.2515	0.1359	0.1087
21	1.180	0.239	3.3695	-0.0017	0.0997
22	1.366	0.123	3.3914	0.0025	0.0994
23	1.566	0.0	3.4190	0.0	0.0987
24	1.022	0.585	3.1504	0.2625	0.1085
25	1.197	0.476	3.2783	0.1371	0.1081
26	1.379	0.362	3.3969	0.0003	0.0991
27	1.567	0.247	3.4188	0.0044	0.0988
28	1.770	0.124	3.4464	0.0019	0.0981
29	1.978	0.0	3.4737	0.0	0.0975
30	1.372	0.614	3.2024	0.2669	0.1075
31	1.583	0.487	3.4231	-0.0002	0.0985
32	1.773	0.372	3.4449	0.0040	0.0982
33	1.980	0.249	3.4725	0.0015	0.0976
34	2.191	0.125	3.4999	-0.0004	0.0969
35	2.406	0.0	3.5259	0.0	0.0964
36	1.839	0.653	3.4079	0.2840	0.0924
37	2.059	0.544	3.5430	0.1173	0.0922
38	2.284	0.430	3.5713	0.1144	0.0917
39	2.514	0.313	3.5994	0.1122	0.0911
40	2.748	0.197	3.6262	0.1123	0.0907
41	3.146	0.0	3.7302	0.0	0.0854
42	2.290	0.691	3.4485	0.2874	0.0923
43	2.530	0.574	3.5942	0.1030	0.0916
44	2.762	0.457	3.6222	0.1008	0.0910
45	2.998	0.339	3.6489	0.1009	0.0906
46	3.415	0.133	3.7535	-0.0111	0.0853
47	3.685	0.0	3.7767	0.0	0.0852
48	2.786	0.732	3.5071	0.2923	0.0911
49	3.039	0.614	3.6538	0.1012	0.0904
50	3.278	0.496	3.6805	0.1013	0.0900
51	3.721	0.281	3.7862	-0.0103	0.0847
52	3.993	0.149	3.8093	0.0007	0.0846
53	4.306	0.0	3.8419	0.0	0.0840
54	3.310	0.776	3.5650	0.2971	0.0900
55	3.569	0.657	3.7121	0.1003	0.0894
56	4.037	0.432	3.8188	-0.0111	0.0841
57	4.312	0.301	3.8418	-0.0001	0.0840
58	4.629	0.152	3.8744	-0.0008	0.0834
59	4.957	0.0	3.9070	0.0	0.0828
60	3.850	0.821	3.6211	0.3018	0.0891
61	4.368	0.587	3.8505	-0.0159	0.0837
62	4.644	0.457	3.8733	-0.0049	0.0836
63	4.965	0.307	3.9060	-0.0056	0.0830
64	5.297	0.155	3.9365	-0.0048	0.0824
65	5.640	0.0	3.9700	0.0	0.0820
66	5.019	0.918	3.8170	0.3181	0.0801
67	5.354	0.787	3.9772	0.0708	0.0800
68	5.699	0.643	4.0106	0.0700	0.0794
69	6.054	0.497	4.0440	0.0707	0.0789
70	6.420	0.349	4.0762	0.0753	0.0786
71	7.289	0.0	4.1872	0.0	0.0753
72	5.736	0.978	3.8690	0.3224	0.0800
73	6.117	0.831	4.0420	0.0502	0.0794
74	6.474	0.683	4.0752	0.0509	0.0789
75	6.841	0.534	4.1072	0.0556	0.0785
76	7.732	0.175	4.2186	-0.0196	0.0752
77	8.173	0.0	4.2496	0.0	0.0751
78	6.561	1.047	3.9419	0.3285	0.0789

TABLE V ANALYTICAL RESULTS FOR SUPERCRITICAL FLOW

MAIN CHANNEL FLOWRATE: 0.3022 CFS LATERAL INFLOW: 0.02360 CFS/FT SLOPE: 1.00 %

POINT	DOWNSTREAM DISTANCE (FT)	TRANSVERSE DISTANCE (FT)	VELOCITY (FPS) X-DIRECTION	VELOCITY (FPS) Y-DIRECTION	DEPTH (FT)
0	0.0	0.500	2.9770	0.0	0.1220
1	0.025	0.478	2.9818	0.0	0.1219
2	0.161	0.359	3.0077	0.0	0.1213
3	0.294	0.239	3.0335	0.0	0.1207
4	0.440	0.120	3.0590	0.0	0.1201
5	0.584	0.000	3.0845	0.0	0.1195
6	0.052	0.504	2.8463	0.2372	0.1179
7	0.197	0.390	3.0554	0.0494	0.1173
8	0.342	0.275	3.0811	0.0490	0.1168
9	0.489	0.160	3.1067	0.0486	0.1163
10	0.640	0.045	3.1321	0.0483	0.1158
11	0.700	0.0	3.1793	0.0	0.1122
12	0.347	0.529	2.8925	0.2410	0.1173
13	0.503	0.409	3.1049	0.0440	0.1166
14	0.653	0.294	3.1303	0.0437	0.1160
15	0.806	0.178	3.1557	0.0434	0.1155
16	0.873	0.128	3.2029	-0.0046	0.1119
17	1.048	0.0	3.2265	0.0	0.1115
18	0.667	0.556	2.9477	0.2456	0.1162
19	0.830	0.435	3.1585	0.0424	0.1155
20	0.986	0.320	3.1837	0.0421	0.1149
21	1.061	0.265	3.2310	-0.0055	0.1113
22	1.239	0.137	3.2545	-0.0010	0.1110
23	1.432	0.0	3.2824	0.0	0.1104
24	1.003	0.584	3.0015	0.2501	0.1152
25	1.171	0.463	3.2114	0.0401	0.1144
26	1.255	0.403	3.2589	-0.0072	0.1107
27	1.435	0.276	3.2822	-0.0027	0.1104
28	1.632	0.139	3.3100	-0.0017	0.1098
29	1.834	0.0	3.3376	0.0	0.1093
30	1.351	0.613	3.0548	0.2546	0.1141
31	1.454	0.542	3.2857	-0.0106	0.1103
32	1.636	0.415	3.3089	-0.0061	0.1100
33	1.835	0.278	3.3367	-0.0051	0.1094
34	2.040	0.139	3.3642	-0.0034	0.1089
35	2.249	0.0	3.3906	0.0	0.1084
36	1.570	0.631	3.1372	0.2614	0.1081
37	1.768	0.505	3.3484	0.0248	0.1077
38	1.973	0.371	3.3762	0.0256	0.1071
39	2.185	0.235	3.4038	0.0271	0.1066
40	2.400	0.099	3.4303	0.0303	0.1062
41	2.558	0.0	3.4703	0.0	0.1041
42	1.983	0.665	3.1839	0.2653	0.1078
43	2.207	0.526	3.4021	0.0165	0.1070
44	2.421	0.389	3.4296	0.0181	0.1065
45	2.636	0.253	3.4560	0.0213	0.1061
46	2.803	0.151	3.4960	-0.0089	0.1039
47	3.049	0.0	3.5216	0.0	0.1037
48	2.456	0.705	3.2438	0.2703	0.1067
49	2.691	0.563	3.4621	0.0134	0.1059
50	2.911	0.427	3.4884	0.0167	0.1055
51	3.084	0.321	3.5266	-0.0133	0.1034
52	3.333	0.171	3.5540	-0.0045	0.1032
53	3.620	0.0	3.5864	0.0	0.1027
54	2.954	0.746	3.3028	0.2752	0.1057
55	3.197	0.604	3.5208	0.0104	0.1051
56	3.378	0.495	3.5612	-0.0194	0.1029
57	3.629	0.345	3.5865	-0.0106	0.1027
58	3.920	0.175	3.6187	-0.0061	0.1022
59	4.223	0.0	3.6509	0.0	0.1017
60	3.470	0.789	3.3601	0.2800	0.1049
61	3.660	0.670	3.5927	-0.0278	0.1025
62	3.932	0.521	3.6178	-0.0190	0.1024
63	4.226	0.350	3.6499	-0.0145	0.1018
64	4.532	0.175	3.6820	-0.0084	0.1014
65	4.844	0.0	3.7129	0.0	0.1010
66	3.926	0.827	3.4354	0.2863	0.1014
67	4.206	0.676	3.6579	-0.0054	0.1010
68	4.507	0.507	3.6900	-0.0010	0.1005
69	4.819	0.334	3.7221	0.0050	0.1001
70	5.137	0.161	3.7530	0.0134	0.0998
71	5.438	0.0	3.7934	0.0	0.0985
72	4.528	0.877	3.4910	0.2909	0.1011
73	4.861	0.702	3.7224	-0.0149	0.1003
74	5.175	0.529	3.7543	-0.0089	0.0999
75	5.495	0.356	3.7851	-0.0005	0.0996
76	5.803	0.192	3.8254	-0.0138	0.0983
77	6.168	0.0	3.8572	0.0	0.0981
78	5.249	0.937	3.5623	0.2969	0.1001
79	5.603	0.757	3.7954	-0.0199	0.0994
80	5.926	0.584	3.8259	-0.0116	0.0991
81	6.242	0.417	3.8663	-0.0248	0.0978
82	6.610	0.226	3.8979	-0.0110	0.0976
83	7.055	0.0	3.9383	0.0	0.0971
84	6.020	1.002	3.6339	0.3028	0.0992

TABLE VI ANALYTICAL RESULTS FOR SUPERCRITICAL FLOW

MAIN CHANNEL FLOWRATE: 0.3632 CFS LATERAL INFLOW: 0.0 CFS/FT SLOPE: 4.00 %

POINT	DOWNSTREAM DISTANCE (FT)	TRANSVERSE DISTANCE (FT)	VELOCITY (FPS) X-DIRECTION	VELOCITY (FPS) Y-DIRECTION	DEPTH (FT)
0	0.0	0.500	3.4923	0.0	0.1040
1	0.036	0.478	3.5105	0.0	0.1036
2	0.237	0.359	3.6089	0.0	0.1015
3	0.449	0.239	3.7071	0.0	0.0995
4	0.671	0.120	3.8053	0.0	0.0975
5	0.904	0.000	3.9034	0.0	0.0955
6	0.083	0.507	3.6880	0.3073	0.0895
7	0.323	0.409	2.7942	0.3038	0.0881
8	0.577	0.311	3.9005	0.3008	0.0868
9	0.842	0.211	4.0069	0.2984	0.0854
10	1.120	0.111	4.1138	0.2962	0.0840
11	1.424	0.0	4.3542	0.0	0.0744
12	0.558	0.546	3.8991	0.3249	0.0858
13	0.825	0.449	4.0062	0.3217	0.0845
14	1.104	0.352	4.1137	0.3189	0.0832
15	1.397	0.253	4.2215	0.3166	0.0819
16	1.764	0.127	4.4746	0.0211	0.0723
17	2.125	0.0	4.5976	0.0	0.0703
18	1.086	0.590	4.1123	0.3427	0.0823
19	1.379	0.494	4.2207	0.3397	0.0811
20	1.686	0.397	4.3296	0.3371	0.0798
21	2.122	0.257	4.5963	0.0425	0.0703
22	2.506	0.130	4.7219	0.0214	0.0683
23	2.910	0.0	4.8489	0.0	0.0665
24	1.667	0.639	4.3283	0.3607	0.0790
25	1.989	0.544	4.4384	0.3579	0.0778
26	2.498	0.389	4.7193	0.0641	0.0684
27	2.906	0.262	4.8476	0.0429	0.0665
28	3.335	0.133	4.9775	0.0216	0.0646
29	3.786	0.0	5.1091	0.0	0.0629
30	2.306	0.692	4.5477	0.3790	0.0759
31	2.894	0.524	4.8437	0.0858	0.0665
32	3.327	0.397	4.9749	0.0647	0.0647
33	3.782	0.268	5.1078	0.0433	0.0629
34	4.260	0.136	5.2426	0.0218	0.0612
35	4.762	0.0	5.3793	0.0	0.0596
36	3.697	0.808	5.1516	0.4293	0.0594
37	4.248	0.705	5.3048	0.4086	0.0578
38	4.831	0.599	5.4607	0.3877	0.0562
39	5.449	0.490	5.6197	0.3667	0.0547
40	6.102	0.377	5.7820	0.3456	0.0533
41	8.140	0.0	6.3202	0.0	0.0473
42	4.815	0.901	5.4575	0.4548	0.0559
43	5.441	0.799	5.6189	0.4339	0.0544
44	6.104	0.693	5.7837	0.4130	0.0530
45	6.807	0.584	5.9520	0.3919	0.0516
46	9.126	0.194	6.5335	0.0454	0.0456
47	10.206	0.0	6.7582	0.0	0.0440
48	6.087	1.007	5.7804	0.4817	0.0527

TABLE VII ANALYTICAL RESULTS FOR SUPERCRITICAL FLOW

MAIN CHANNEL FLOWRATE: 0.3632 CFS LATERAL INFLOW: 0.01250 CFS/FT SLOPE: 4.00 %

POINT	DOWNSTREAM DISTANCE (FT)	TRANSVERSE DISTANCE (FT)	VELOCITY (FPS) X-DIRECTION	VELOCITY (FPS) Y-DIRECTION	DEPTH (FT)
0	0.0	0.500	3.4923	0.0	0.1040
1	0.036	0.478	3.5105	0.0	0.1036
2	0.237	0.359	3.6089	0.0	0.1015
3	0.449	0.239	3.7071	0.0	0.0995
4	0.671	0.120	3.8053	0.0	0.0975
5	0.904	0.000	3.9034	0.0	0.0955
6	0.080	0.507	3.4984	0.2915	0.0968
7	0.305	0.398	3.7083	0.1275	0.0951
8	0.536	0.287	3.8096	0.1262	0.0934
9	0.778	0.176	3.9110	0.1251	0.0917
10	1.031	0.064	4.0126	0.1242	0.0900
11	1.180	0.0	4.1276	0.0	0.0850
12	0.552	0.546	3.6997	0.3083	0.0934
13	0.808	0.434	3.9189	0.1180	0.0916
14	1.061	0.322	4.0203	0.1169	0.0900
15	1.326	0.210	4.1219	0.1160	0.0883
16	1.501	0.137	4.2412	-0.0075	0.0832
17	1.835	0.0	4.3548	0.0	0.0815
18	1.096	0.591	3.9114	0.3260	0.0899
19	1.380	0.478	4.1377	0.1072	0.0881
20	1.657	0.365	4.2394	0.1063	0.0865
21	1.862	0.283	4.3634	-0.0166	0.0814
22	2.209	0.146	4.4771	-0.0090	0.0797
23	2.598	0.0	4.5995	0.0	0.0780
24	1.707	0.642	4.1267	0.3439	0.0865
25	2.021	0.528	4.3617	0.0917	0.0848
26	2.259	0.436	4.4909	-0.0305	0.0796
27	2.620	0.298	4.6045	-0.0230	0.0780
28	3.023	0.152	4.7268	-0.0139	0.0764
29	3.462	0.0	4.8540	0.0	0.0748
30	2.392	0.699	4.3472	0.3623	0.0833
31	2.696	0.594	4.6234	-0.0520	0.0780
32	3.070	0.456	4.7368	-0.0445	0.0764
33	3.487	0.309	4.8588	-0.0355	0.0748
34	3.940	0.157	4.9855	-0.0215	0.0733
35	4.430	0.0	5.1162	0.0	0.0718
36	3.112	0.759	4.6158	0.3847	0.0751
37	3.568	0.623	4.9020	0.0042	0.0734
38	4.014	0.480	5.0267	0.0132	0.0719
39	4.496	0.332	5.1559	0.0271	0.0705
40	5.016	0.180	5.2889	0.0487	0.0692
41	5.657	0.0	5.4689	0.0	0.0666
42	4.175	0.848	4.8918	0.4076	0.0721
43	4.724	0.697	5.2049	-0.0422	0.0703
44	5.218	0.547	5.3324	-0.0284	0.0689
45	5.749	0.394	5.4632	-0.0068	0.0677
46	6.426	0.205	5.6448	-0.0557	0.0651
47	7.199	0.0	5.8160	0.0	0.0638
48	5.499	0.958	5.2059	0.4338	0.0690
49	6.141	0.796	5.5449	-0.1070	0.0672
50	6.681	0.640	5.6723	-0.0855	0.0660
51	7.399	0.439	5.8553	-0.1348	0.0635
52	8.171	0.234	6.0200	-0.0791	0.0623
53	9.131	0.0	6.2153	0.0	0.0610
54	7.122	1.094	5.5552	0.4629	0.0659

TABLE VIII ANALYTICAL RESULTS FOR SUPERCRITICAL FLOW

MAIN CHANNEL FLOWRATE: 0.3632 CFS LATERAL INFLOW: 0.02360 CFS/FT SLOPE: 4.00 %

POINT	DOWNSTREAM DISTANCE (FT)	TRANSVERSE DISTANCE (FT)	VELOCITY (FPS) X-DIRECTION	VELOCITY (FPS) Y-DIRECTION	DEPTH (FT)
0	0.0	0.500	3.4923	0.0	0.1040
1	0.036	0.476	3.5105	0.0	0.1036
2	0.237	0.359	3.6089	0.0	0.1015
3	0.449	0.239	3.7071	0.0	0.0995
4	0.671	0.120	3.8053	0.0	0.0975
5	0.904	0.000	3.9034	0.0	0.0955
6	0.078	0.507	3.3385	0.2782	0.1034
7	0.290	0.389	3.6323	-0.0027	0.1011
8	0.504	0.269	3.7305	-0.0027	0.0991
9	0.729	0.150	3.8286	-0.0026	0.0971
10	0.964	0.030	3.9267	-0.0026	0.0952
11	1.024	0.0	3.9499	0.0	0.0948
12	0.542	0.545	3.5440	0.2953	0.0997
13	0.791	0.420	3.8458	-0.0205	0.0973
14	1.026	0.300	3.9435	-0.0203	0.0953
15	1.272	0.179	4.0413	-0.0201	0.0934
16	1.335	0.149	4.0644	-0.0175	0.0931
17	1.656	0.0	4.1780	0.0	0.0914
18	1.095	0.591	3.7548	0.3129	0.0961
19	1.373	0.464	4.0662	-0.0505	0.0938
20	1.630	0.341	4.1635	-0.0502	0.0919
21	1.695	0.311	4.1865	-0.0476	0.0916
22	2.026	0.161	4.2989	-0.0300	0.0900
23	2.405	0.0	4.4182	0.0	0.0886
24	1.726	0.644	3.9701	0.3308	0.0928
25	2.036	0.513	4.2935	-0.0890	0.0905
26	2.103	0.483	4.3162	-0.0863	0.0901
27	2.444	0.332	4.4273	-0.0687	0.0886
28	2.831	0.170	4.5447	-0.0387	0.0872
29	3.264	0.0	4.6690	0.0	0.0859
30	2.446	0.704	4.1917	0.3493	0.0895
31	2.557	0.660	4.4513	-0.1345	0.0888
32	2.907	0.508	4.5607	-0.1168	0.0872
33	3.300	0.345	4.6759	-0.0868	0.0859
34	3.738	0.175	4.7977	-0.0481	0.0847
35	4.216	0.0	4.9235	0.0	0.0836
36	2.706	0.725	4.2559	0.3547	0.0889
37	3.096	0.576	4.6086	-0.1486	0.0870
38	3.490	0.412	4.7226	-0.1185	0.0858
39	3.927	0.241	4.8429	-0.0798	0.0846
40	4.403	0.065	4.9670	-0.0317	0.0834
41	4.589	0.0	5.0098	0.0	0.0833
42	3.648	0.804	4.5164	0.3764	0.0859
43	4.145	0.629	4.8951	-0.1840	0.0841
44	4.588	0.457	5.0121	-0.1453	0.0830
45	5.066	0.282	5.1323	-0.0972	0.0820
46	5.248	0.218	5.1730	-0.0656	0.0819
47	5.909	0.0	5.3310	0.0	0.0806
48	4.882	0.907	4.8126	0.4010	0.0833
49	5.469	0.715	5.2171	-0.2554	0.0815
50	5.943	0.539	5.3316	-0.2074	0.0806
51	6.119	0.477	5.3693	-0.1757	0.0806
52	6.770	0.257	5.5196	-0.1102	0.0794
53	7.608	0.0	5.6972	0.0	0.0784
54	6.406	1.034	5.1425	0.4285	0.0807

TABLE IX ANALYTICAL RESULTS FOR SUPERCRITICAL FLOW

MAIN CHANNEL FLOWRATE: 0.1967 CFS LATERAL INFLOW: 0.0 CFS/FT SLOPE: 1.00 %

POINT	DOWNSTREAM DISTANCE (FT)	TRANSVERSE DISTANCE (FT)	VELOCITY (FPS) X-DIRECTION	VELOCITY (FPS) Y-DIRECTION	DEPTH (FT)
0	0.0	0.500	2.8097	0.0	0.0700
1	0.035	0.478	2.8153	0.0	0.0699
2	0.227	0.359	2.8456	0.0	0.0694
3	0.422	0.239	2.8758	0.0	0.0688
4	0.622	0.120	2.9059	0.0	0.0683
5	0.825	0.000	2.9361	0.0	0.0678
6	0.080	0.507	2.9494	0.2458	0.0605
7	0.308	0.409	2.9830	0.2424	0.0602
8	0.539	0.310	3.0153	0.2412	0.0598
9	0.776	0.211	3.0476	0.2400	0.0595
10	1.017	0.112	3.0799	0.2389	0.0591
11	1.284	0.0	3.2349	0.0	0.0521
12	0.522	0.544	3.0160	0.2513	0.0595
13	0.760	0.446	3.0486	0.2502	0.0591
14	1.002	0.349	3.0811	0.2491	0.0588
15	1.249	0.251	3.1137	0.2480	0.0585
16	1.566	0.123	3.2727	0.0113	0.0514
17	1.857	0.0	3.3111	0.0	0.0507
18	0.983	0.582	3.0806	0.2567	0.0585
19	1.230	0.485	3.1132	0.2556	0.0582
20	1.482	0.387	3.1458	0.2545	0.0579
21	1.847	0.244	3.3080	0.0185	0.0509
22	2.148	0.121	3.3469	0.0070	0.0502
23	2.445	0.0	3.3829	0.0	0.0496
24	1.463	0.622	3.1452	0.2621	0.0576
25	1.720	0.525	3.1780	0.2610	0.0573
26	2.135	0.366	3.3434	0.0256	0.0503
27	2.444	0.242	3.3827	0.0140	0.0496
28	2.749	0.122	3.4190	0.0070	0.0490
29	3.061	0.0	3.4554	0.0	0.0485
30	1.962	0.664	3.2102	0.2675	0.0567
31	2.428	0.489	3.3789	0.0328	0.0497
32	2.747	0.365	3.4187	0.0210	0.0490
33	3.060	0.244	3.4553	0.0140	0.0485
34	3.380	0.123	3.4919	0.0070	0.0480
35	3.706	0.0	3.5288	0.0	0.0474
36	3.037	0.753	3.5284	0.2940	0.0444
37	3.425	0.648	3.5743	0.2797	0.0438
38	3.803	0.548	3.6154	0.2723	0.0434
39	4.190	0.446	3.6566	0.2650	0.0429
40	4.586	0.342	3.6981	0.2576	0.0425
41	5.828	0.0	3.9031	0.0	0.0382
42	3.809	0.817	3.6185	0.3015	0.0431
43	4.202	0.719	3.6604	0.2943	0.0427
44	4.603	0.619	3.7024	0.2871	0.0423
45	5.015	0.518	3.7448	0.2799	0.0418
46	6.380	0.165	3.9592	0.0252	0.0376
47	6.965	0.0	4.0175	0.0	0.0369
48	4.584	0.882	3.7009	0.3084	0.0421
49	4.997	0.783	3.7435	0.3012	0.0417
50	5.419	0.683	3.7863	0.2940	0.0413
51	6.890	0.317	4.0076	0.0397	0.0370
52	7.501	0.150	4.0674	0.0142	0.0363
53	8.056	0.0	4.1181	0.0	0.0358
54	5.399	0.950	3.7848	0.3154	0.0412
55	5.834	0.851	3.8282	0.3092	0.0408
56	7.414	0.471	4.0564	0.0541	0.0365
57	8.052	0.304	4.1177	0.0284	0.0358
58	8.626	0.154	4.1694	0.0142	0.0353
59	9.217	0.0	4.2215	0.0	0.0348
60	6.259	1.022	3.8702	0.3225	0.0402

TABLE X ANALYTICAL RESULTS FOR SUPERCRITICAL FLOW

MAIN CHANNEL FLOWRATE: 0.1967 CFS LATERAL INFLOW: 0.01250 CFS/FT SLOPE: 1.00 %

POINT	DOWNSTREAM DISTANCE (FT)	TRANSVERSE DISTANCE (FT)	VELOCITY (FPS) X-DIRECTION	VELOCITY (FPS) Y-DIRECTION	DEPTH (FT)
0	0.0	0.500	2.8097	0.0	0.0700
1	0.035	0.476	2.8153	0.0	0.0699
2	0.227	0.359	2.8456	0.0	0.0694
3	0.422	0.239	2.8758	0.0	0.0688
4	0.622	0.120	2.9059	0.0	0.0683
5	0.825	0.000	2.9361	0.0	0.0678
6	0.075	0.506	2.6773	0.2231	0.0694
7	0.278	0.390	2.8594	0.0096	0.0688
8	0.476	0.271	2.8897	0.0097	0.0683
9	0.678	0.153	2.9200	0.0096	0.0678
10	0.884	0.034	2.9502	0.0096	0.0673
11	0.944	0.0	2.9644	0.0	0.0667
12	0.503	0.542	2.7362	0.2280	0.0686
13	0.725	0.420	2.9224	0.0022	0.0679
14	0.931	0.300	2.9527	0.0021	0.0674
15	1.140	0.181	2.9828	0.0021	0.0669
16	1.203	0.146	2.9970	-0.0074	0.0664
17	1.465	0.0	3.0295	0.0	0.0661
18	0.974	0.581	2.8021	0.2335	0.0676
19	1.207	0.458	2.9892	-0.0030	0.0670
20	1.421	0.338	3.0194	-0.0031	0.0664
21	1.487	0.301	3.0337	-0.0126	0.0659
22	1.753	0.156	3.0660	-0.0052	0.0656
23	2.046	0.0	3.1024	0.0	0.0651
24	1.472	0.623	2.8668	0.2389	0.0667
25	1.715	0.499	3.0559	-0.0102	0.0660
26	1.785	0.461	3.0704	-0.0197	0.0655
27	2.054	0.315	3.1025	-0.0123	0.0652
28	2.351	0.159	3.1388	-0.0071	0.0647
29	2.661	0.0	3.1750	0.0	0.0643
30	1.995	0.666	2.9318	0.2443	0.0658
31	2.089	0.620	3.1067	-0.0275	0.0651
32	2.362	0.474	3.1388	-0.0202	0.0648
33	2.663	0.318	3.1749	-0.0150	0.0643
34	2.977	0.159	3.2110	-0.0079	0.0639
35	3.296	0.0	3.2467	0.0	0.0635
36	2.198	0.683	2.9616	0.2468	0.0649
37	2.494	0.541	3.1565	-0.0187	0.0644
38	2.798	0.385	3.1927	-0.0136	0.0640
39	3.114	0.226	3.2287	-0.0065	0.0636
40	3.436	0.068	3.2645	0.0013	0.0632
41	3.577	0.0	3.2822	0.0	0.0629
42	2.839	0.737	3.0313	0.2526	0.0643
43	3.186	0.576	3.2339	-0.0279	0.0637
44	3.506	0.417	3.2698	-0.0209	0.0633
45	3.832	0.256	3.3053	-0.0130	0.0629
46	3.976	0.189	3.3230	-0.0143	0.0626
47	4.377	0.0	3.3634	0.0	0.0623
48	3.598	0.800	3.1133	0.2594	0.0634
49	3.970	0.635	3.3183	-0.0352	0.0628
50	4.301	0.475	3.3537	-0.0275	0.0625
51	4.448	0.406	3.3714	-0.0268	0.0622
52	4.852	0.217	3.4114	-0.0145	0.0619
53	5.333	0.0	3.4590	0.0	0.0614
54	4.417	0.868	3.1956	0.2663	0.0626
55	4.805	0.702	3.4034	-0.0454	0.0621
56	4.956	0.631	3.4212	-0.0466	0.0618
57	5.364	0.443	3.4607	-0.0325	0.0615
58	5.848	0.225	3.5079	-0.0180	0.0611
59	6.366	0.0	3.5561	0.0	0.0607
60	5.278	0.940	3.2783	0.2732	0.0619
61	5.479	0.856	3.4706	-0.0662	0.0614
62	5.891	0.667	3.5099	-0.0520	0.0611
63	6.378	0.449	3.5566	-0.0376	0.0607
64	6.899	0.224	3.6043	-0.0195	0.0604
65	7.434	0.0	3.6519	0.0	0.0600
66	5.728	0.577	3.3219	0.2768	0.0613
67	6.189	0.791	3.5383	-0.0607	0.0609
68	6.680	0.573	3.5849	-0.0463	0.0605
69	7.203	0.348	3.6323	-0.0282	0.0601
70	7.740	0.125	3.6796	-0.0087	0.0598
71	8.047	0.0	3.7072	0.0	0.0596
72	6.766	1.064	3.4132	0.2844	0.0607

TABLE XI ANALYTICAL RESULTS FOR SUPERCRITICAL FLOW

MAIN CHANNEL FLOWRATE: 0.1967 CFS LATERAL INFLOW: 0.02360 CFS/FT SLOPE: 1.00 %

POINT	DOWNSTREAM DISTANCE (FT)	TRANSVERSE DISTANCE (FT)	VELOCITY (FPS) X-DIRECTION	VELOCITY (FPS) Y-DIRECTION	DEPTH (FT)
0	0.0	0.500	2.8097	0.0	0.0700
1	0.210	0.369	2.8430	0.0	0.0694
2	0.361	0.276	2.8663	0.0	0.0690
3	0.513	0.184	2.8896	0.0	0.0686
4	0.668	0.092	2.9128	0.0	0.0682
5	0.825	0.0	2.9361	0.0	0.0678
6	0.438	0.537	2.5072	0.2089	0.0767
7	0.605	0.423	2.8055	-0.1523	0.0753
8	0.744	0.320	2.8264	-0.1492	0.0748
9	0.884	0.217	2.8488	-0.1487	0.0742
10	1.027	0.115	2.8711	-0.1482	0.0737
11	1.191	0.0	2.8049	0.0	0.0805
12	0.805	0.567	2.5880	0.2157	0.0742
13	0.979	0.455	2.8689	-0.1416	0.0737
14	1.125	0.353	2.8916	-0.1416	0.0732
15	1.272	0.252	2.9140	-0.1411	0.0727
16	1.415	0.155	2.8464	0.0058	0.0795
17	1.647	0.0	2.8880	0.0	0.0785
18	1.185	0.599	2.6339	0.2195	0.0737
19	1.365	0.486	2.9186	-0.1556	0.0731
20	1.514	0.384	2.9410	-0.1552	0.0726
21	1.634	0.303	2.8722	-0.0092	0.0796
22	1.670	0.145	2.9136	-0.0149	0.0786
23	2.093	0.0	2.9368	0.0	0.0786
24	1.589	0.632	2.6691	0.2224	0.0739
25	1.771	0.520	2.9635	-0.1782	0.0729
26	1.866	0.455	2.8933	-0.0332	0.0801
27	2.105	0.295	2.9345	-0.0387	0.0790
28	2.326	0.146	2.9543	-0.0238	0.0791
29	2.557	0.0	2.9794	0.0	0.0796
30	2.005	0.667	2.7213	0.2268	0.0731
31	2.103	0.607	2.9271	-0.0385	0.0797
32	2.347	0.445	2.9682	-0.0440	0.0786
33	2.571	0.300	2.9929	-0.0292	0.0787
34	2.801	0.154	3.0127	-0.0055	0.0792
35	3.050	0.0	3.0457	0.0	0.0787
36	2.204	0.684	2.6534	0.2211	0.0829
37	2.460	0.514	2.9424	-0.1089	0.0814
38	2.674	0.363	2.9660	-0.0929	0.0815
39	2.895	0.211	2.9852	-0.0689	0.0820
40	3.133	0.051	3.0177	-0.0630	0.0815
41	3.210	0.0	2.9895	0.0	0.0844
42	2.765	0.730	2.7493	0.2291	0.0806
43	3.021	0.571	3.0179	-0.0931	0.0805
44	3.243	0.423	3.0371	-0.0698	0.0810
45	3.488	0.263	3.0694	-0.0639	0.0805
46	3.552	0.222	3.0404	-0.0015	0.0835
47	3.904	0.0	3.0911	0.0	0.0826
48	3.319	0.777	2.8096	0.2341	0.0807
49	3.579	0.619	3.0730	-0.0892	0.0810
50	3.826	0.458	3.1052	-0.0835	0.0805
51	3.876	0.426	3.0755	-0.0215	0.0835
52	4.233	0.201	3.1258	-0.0199	0.0826
53	4.563	0.0	3.1600	0.0	0.0826
54	3.879	0.823	2.8642	0.2387	0.0813
55	4.170	0.651	3.1385	-0.1064	0.0806
56	4.206	0.628	3.1082	-0.0451	0.0838
57	4.567	0.400	3.1581	-0.0432	0.0828
58	4.896	0.198	3.1918	-0.0233	0.0828
59	5.228	0.0	3.2230	0.0	0.0830
60	4.516	0.876	2.9397	0.2450	0.0805
61	4.571	0.844	3.1516	-0.0578	0.0834
62	4.940	0.614	3.2013	-0.0560	0.0824
63	5.271	0.414	3.2346	-0.0362	0.0824
64	5.603	0.218	3.2653	-0.0130	0.0827
65	5.983	0.0	3.3072	0.0	0.0823
66	4.631	0.886	2.9052	0.2421	0.0851
67	5.017	0.658	3.1892	-0.0927	0.0839
68	5.340	0.453	3.2218	-0.0723	0.0839
69	5.665	0.253	3.2522	-0.0489	0.0841
70	6.037	0.031	3.2936	-0.0358	0.0837
71	6.090	0.0	3.2800	0.0	0.0852
72	5.480	0.957	3.0106	0.2509	0.0835
73	5.863	0.742	3.2819	-0.0868	0.0832
74	6.191	0.544	3.3119	-0.0634	0.0834
75	6.569	0.323	3.3529	-0.0507	0.0830
76	6.611	0.298	3.3387	-0.0152	0.0845
77	7.140	0.0	3.3967	0.0	0.0839
78	6.317	1.026	3.0967	0.2581	0.0832

TABLE XII ANALYTICAL RESULTS FOR SUPERCRITICAL FLOW

MAIN CHANNEL FLOWRATE: 0.1967 CFS LATERAL INFLOW: 0.0 CFS/FT SLOPE: 4.00 %

POINT	DOWNSTREAM DISTANCE (FT)	TRANSVERSE DISTANCE (FT)	VELOCITY (FPS) X-DIRECTION	VELOCITY (FPS) Y-DIRECTION	DEPTH (FT)
0	0.0	0.500	3.3336	0.0	0.0590
1	0.049	0.478	3.3561	0.0	0.0587
2	0.323	0.359	3.4791	0.0	0.0569
3	0.615	0.239	3.6029	0.0	0.0552
4	0.924	0.120	3.7275	0.0	0.0536
5	1.250	0.000	3.8531	0.0	0.0521
6	0.116	0.510	3.4934	0.2911	0.0517
7	0.445	0.412	3.6316	0.2869	0.0504
8	0.795	0.314	3.7699	0.2853	0.0492
9	1.168	0.216	3.9097	0.2840	0.0479
10	1.563	0.117	4.0511	0.2830	0.0466
11	2.012	0.0	4.2954	0.0	0.0419
12	0.781	0.565	3.7709	0.3142	0.0485
13	1.158	0.470	3.9126	0.3128	0.0473
14	1.558	0.375	4.0559	0.3118	0.0461
15	1.983	0.279	4.2009	0.3112	0.0449
16	2.546	0.148	4.4715	0.0318	0.0401
17	3.137	0.0	4.6572	0.0	0.0384
18	1.540	0.628	4.0548	0.3379	0.0456
19	1.968	0.536	4.2010	0.3366	0.0445
20	2.421	0.443	4.3491	0.3360	0.0434
21	3.104	0.297	4.6450	0.0575	0.0386
22	3.753	0.148	4.8393	0.0247	0.0369
23	4.423	0.0	5.0285	0.0	0.0355
24	2.403	0.700	4.3480	0.3623	0.0429
25	2.885	0.610	4.4993	0.3615	0.0419
26	3.702	0.452	4.8222	0.0838	0.0371
27	4.414	0.302	5.0259	0.0500	0.0355
28	5.143	0.154	5.2226	0.0252	0.0341
29	5.927	0.0	5.4245	0.0	0.0328
30	3.378	0.781	4.6517	0.3876	0.0405
31	4.341	0.612	5.0034	0.1110	0.0357
32	5.124	0.461	5.2173	0.0759	0.0342
33	5.917	0.313	5.4219	0.0511	0.0328
34	6.769	0.160	5.6323	0.0259	0.0316
35	7.687	0.0	5.8490	0.0	0.0304
36	5.751	0.979	5.4165	0.4514	0.0318
37	6.858	0.860	5.6899	0.4148	0.0304
38	7.960	0.743	5.9461	0.3877	0.0291
39	9.156	0.618	6.2107	0.3604	0.0279
40	10.460	0.486	6.4855	0.3317	0.0268
41	15.047	0.0	7.4009	0.0	0.0231
42	8.127	1.177	5.9851	0.4988	0.0287

TABLE XIII ANALYTICAL RESULTS FOR SUPERCRITICAL FLOW

MAIN CHANNEL FLOWRATE: 0.1967 CFS LATERAL INFLOW: 0.01250 CFS/FT SLOPE: 4.00 %

POINT	DOWNSTREAM DISTANCE (FT)	TRANSVERSE DISTANCE (FT)	VELOCITY (FPS) X-DIRECTION	VELOCITY (FPS) Y-DIRECTION	DEPTH (FT)
0	0.0	0.500	3.3336	0.0	0.0590
1	0.049	0.478	3.3561	0.0	0.0587
2	0.323	0.359	3.4791	0.0	0.0569
3	0.615	0.239	3.6029	0.0	0.0552
4	0.924	0.120	3.7275	0.0	0.0536
5	1.250	0.000	3.8531	0.0	0.0521
6	0.109	0.509	2.1760	0.2647	0.0604
7	0.399	0.390	3.4794	-0.0768	0.0583
8	0.682	0.265	3.5997	-0.0757	0.0565
9	0.982	0.141	3.7210	-0.0755	0.0548
10	1.297	0.017	3.8431	-0.0754	0.0532
11	1.341	0.0	3.8331	0.0	0.0543
12	0.784	0.565	3.4375	0.2865	0.0572
13	1.146	0.433	3.7645	-0.1256	0.0551
14	1.463	0.304	3.8860	-0.1261	0.0534
15	1.794	0.177	4.0071	-0.1258	0.0518
16	1.820	0.167	3.9923	-0.0505	0.0530
17	2.306	0.0	4.1468	0.0	0.0519
18	1.658	0.638	3.7272	0.3106	0.0541
19	2.093	0.496	4.0814	-0.2069	0.0520
20	2.445	0.361	4.2026	-0.2074	0.0504
21	2.446	0.361	4.1622	-0.1320	0.0517
22	2.933	0.193	4.3298	-0.0816	0.0507
23	3.553	0.0	4.5033	0.0	0.0497
24	2.768	0.731	4.0478	0.3373	0.0512
25	3.298	0.565	4.4347	-0.3303	0.0490
26	3.268	0.577	4.4081	-0.2550	0.0504
27	3.748	0.407	4.5462	-0.2047	0.0495
28	4.348	0.214	4.7067	-0.1231	0.0487
29	5.104	0.0	4.8929	0.0	0.0479
30	4.290	0.857	4.4189	0.3662	0.0483
31	4.413	0.823	4.6883	-0.4470	0.0491
32	4.891	0.645	4.8185	-0.3952	0.0482
33	5.449	0.453	4.9606	-0.3136	0.0476
34	6.136	0.241	5.1230	-0.1905	0.0471
35	7.058	0.0	5.3215	0.0	0.0466
36	4.646	0.887	4.4588	0.3716	0.0498
37	5.259	0.706	4.8761	-0.5229	0.0485
38	5.787	0.513	5.0102	-0.4422	0.0479
39	6.432	0.299	5.1628	-0.3203	0.0475
40	7.293	0.053	5.3487	-0.1314	0.0471
41	7.505	0.0	5.3729	0.0	0.0477
42	6.676	1.056	4.9130	0.4094	0.0472

TABLE XIV ANALYTICAL RESULTS FOR SUPERCRITICAL FLOW

MAIN CHANNEL FLOWRATE: 0.1967 CFS LATERAL INFLOW: 0.02360 CFS/FT SLOPE: 4.00 %

POINT	DOWNSTREAM DISTANCE (FT)	TRANSVERSE DISTANCE (FT)	VELOCITY (FPS) X-DIRECTION	VELOCITY (FPS) Y-DIRECTION	DEPTH (FT)
0	0.0	0.500	3.3336	0.0	0.0590
1	0.307	0.366	3.4719	0.0	0.0570
2	0.527	0.274	3.5664	0.0	0.0557
3	0.758	0.183	3.6614	0.0	0.0545
4	0.999	0.091	3.7570	0.0	0.0533
5	1.250	0.000	3.8531	0.0	0.0521
6	0.693	0.558	3.1602	0.2634	0.0651
7	0.959	0.428	3.5694	-0.3738	0.0620
8	1.158	0.318	3.6518	-0.3700	0.0604
9	1.362	0.209	3.7347	-0.3695	0.0589
10	1.574	0.101	3.8178	-0.3693	0.0575
11	1.798	0.0	3.7797	0.0	0.0642
12	1.408	0.617	3.4026	0.2836	0.0616
13	1.730	0.474	3.8285	-0.4667	0.0592
14	1.946	0.361	3.9107	-0.4661	0.0577
15	2.169	0.249	3.9928	-0.4656	0.0563
16	2.293	0.193	3.9304	-0.1000	0.0638
17	2.778	0.0	4.0724	0.0	0.0634
18	2.324	0.694	3.6565	0.3047	0.0593
19	2.724	0.525	4.1133	-0.6197	0.0568
20	2.954	0.407	4.1930	-0.6192	0.0554
21	2.946	0.410	4.1029	-0.2554	0.0637
22	3.405	0.216	4.2326	-0.1551	0.0635
23	3.984	0.0	4.3759	0.0	0.0638
24	3.608	0.801	3.9389	0.3282	0.0577
25	4.163	0.573	4.4517	-0.8861	0.0546
26	3.964	0.675	4.3294	-0.5232	0.0640
27	4.372	0.482	4.4406	-0.4220	0.0641
28	4.867	0.270	4.5595	-0.2670	0.0648
29	5.611	0.0	4.7093	0.0	0.0664
30	5.794	0.983	4.3950	0.3662	0.0545
31	5.806	0.978	4.7203	-0.8981	0.0627
32	6.148	0.799	4.8096	-0.7937	0.0630
33	6.516	0.621	4.8943	-0.6385	0.0644
34	7.050	0.392	4.9948	-0.3713	0.0670
35	8.192	0.0	5.2118	0.0	0.0687
36	5.837	0.986	4.3400	0.3617	0.0640
37	6.277	0.816	4.8045	-0.8697	0.0637
38	6.633	0.640	4.8922	-0.7162	0.0652
39	7.147	0.414	4.9901	-0.4519	0.0679
40	8.251	0.017	5.2023	-0.0839	0.0697
41	8.307	0.0	5.1929	0.0	0.0708
42	7.462	1.122	4.7367	0.3947	0.0617

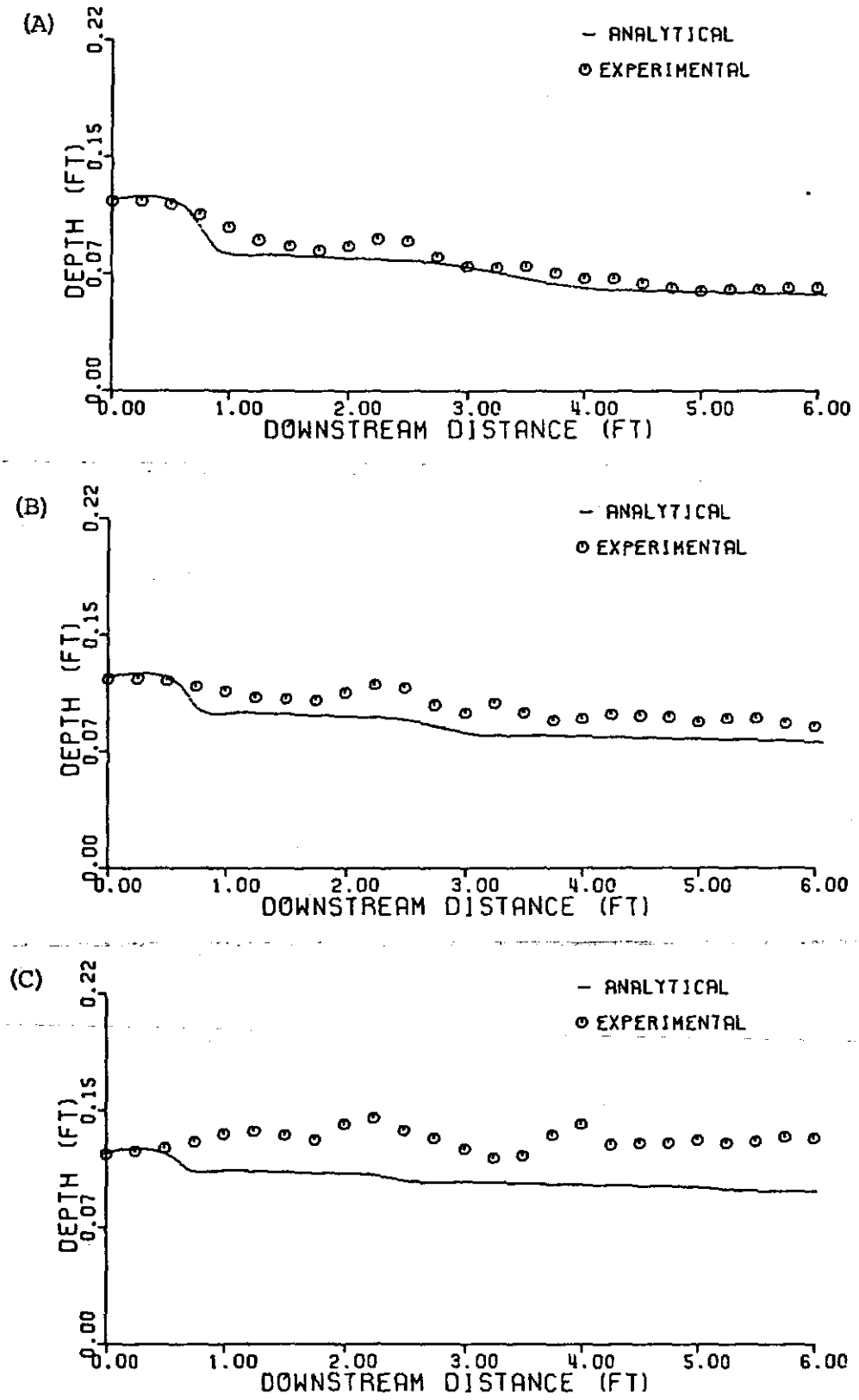


Fig. 12 Relationship Between Depth and Downstream Distance for Supercritical Flow; Q = 0.3632 cfs, S_o = 1%: (A) q_* = 0.0 cfs/ft (B) q_* = 0.0125 cfs/ft (C) q_* = 0.0236 cfs/ft

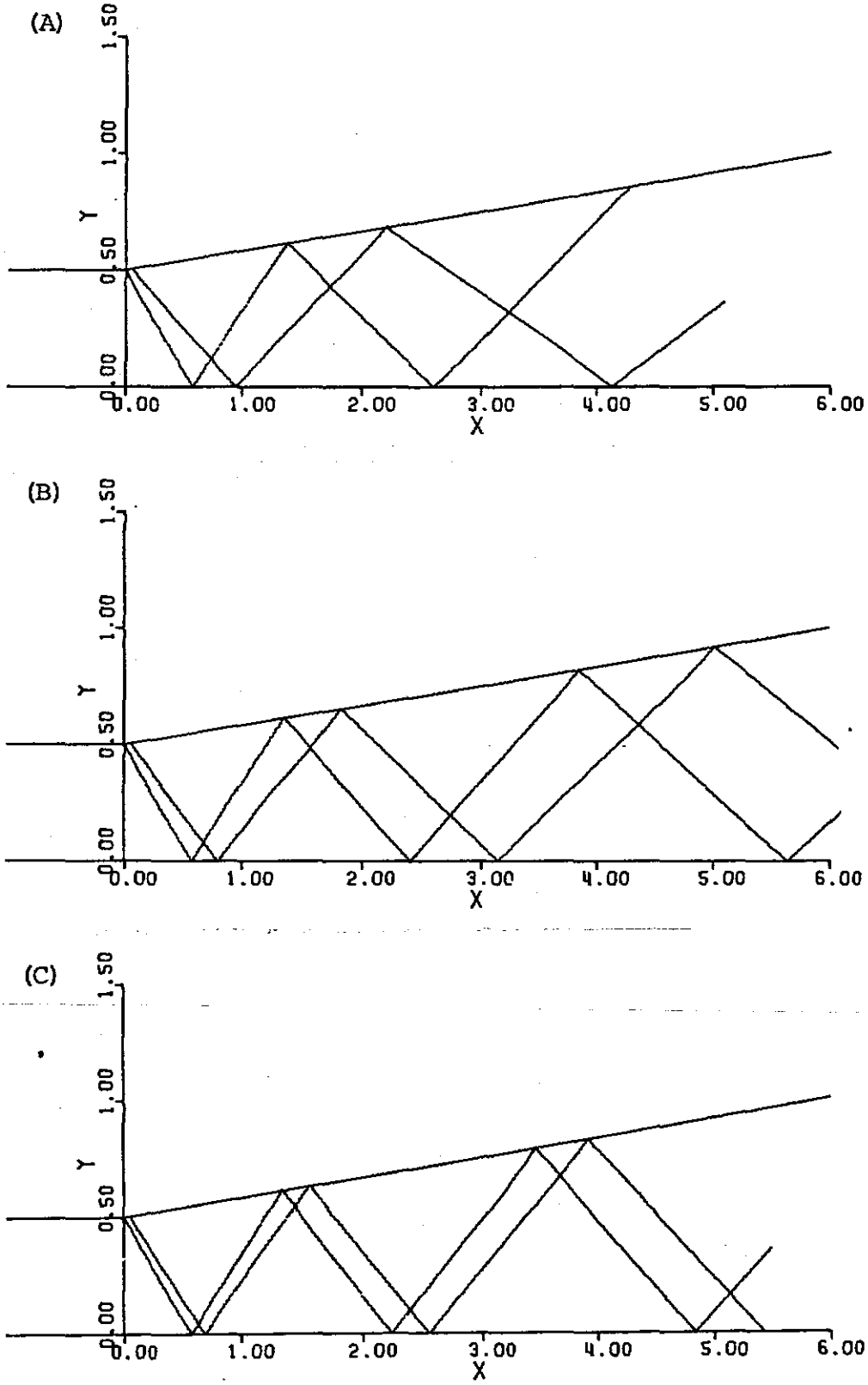


Fig. 13 Cross Wave Patterns for Supercritical Flow; Q = 0.3632 cfs, S_o = 1%: (A) q_* = 0.0 cfs/ft (B) q_* = 0.0125 cfs/ft (C) q_* = 0.0236 cfs/ft

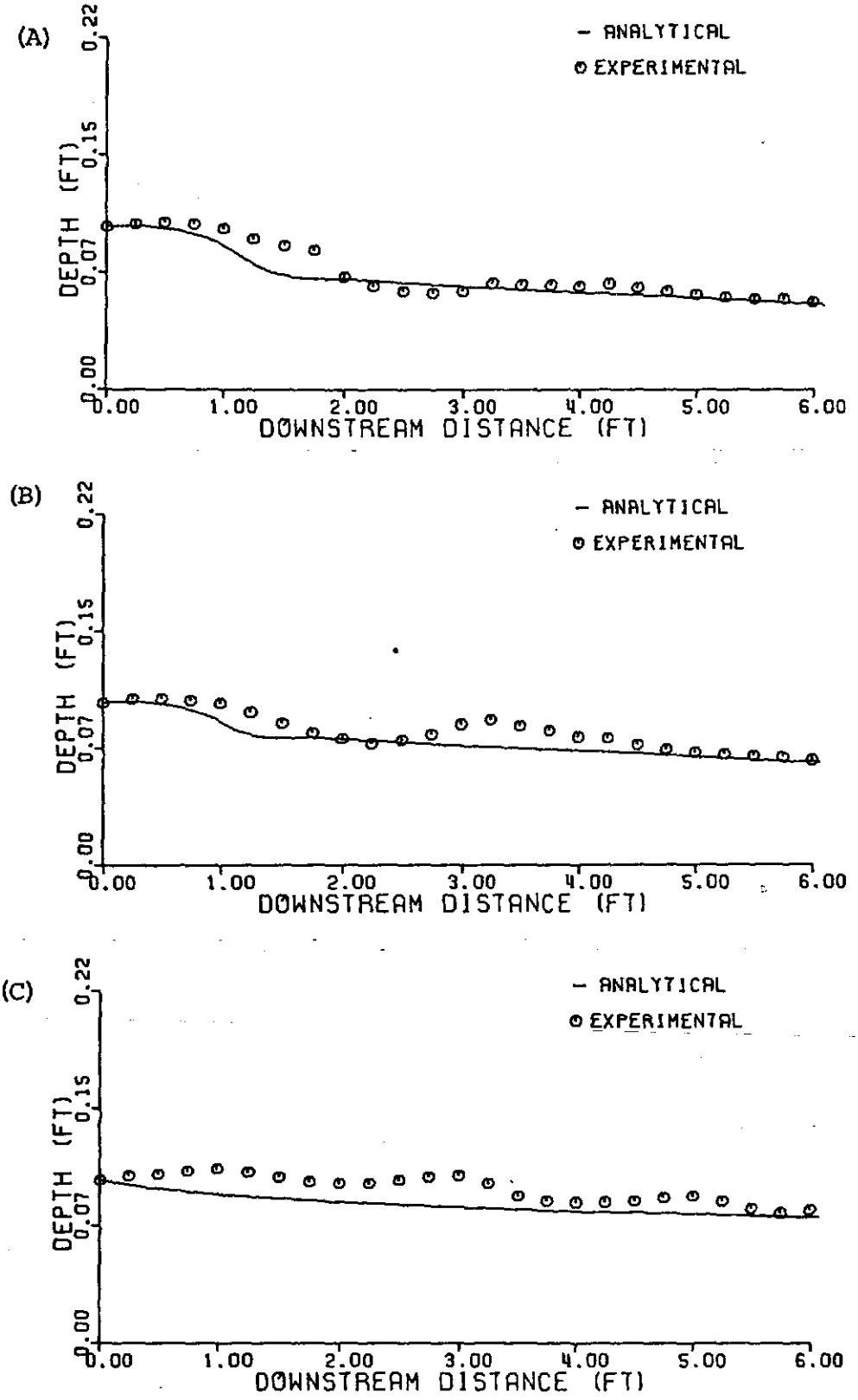


Fig. 14 Relationship Between Depth and Downstream Distance for Supercritical Flow; Q = 0.3632 cfs, S_o = 4%: (A) q_* = 0.0 cfs/ft (B) q_* = 0.0125 cfs/ft (C) q_* = 0.0236 cfs/ft

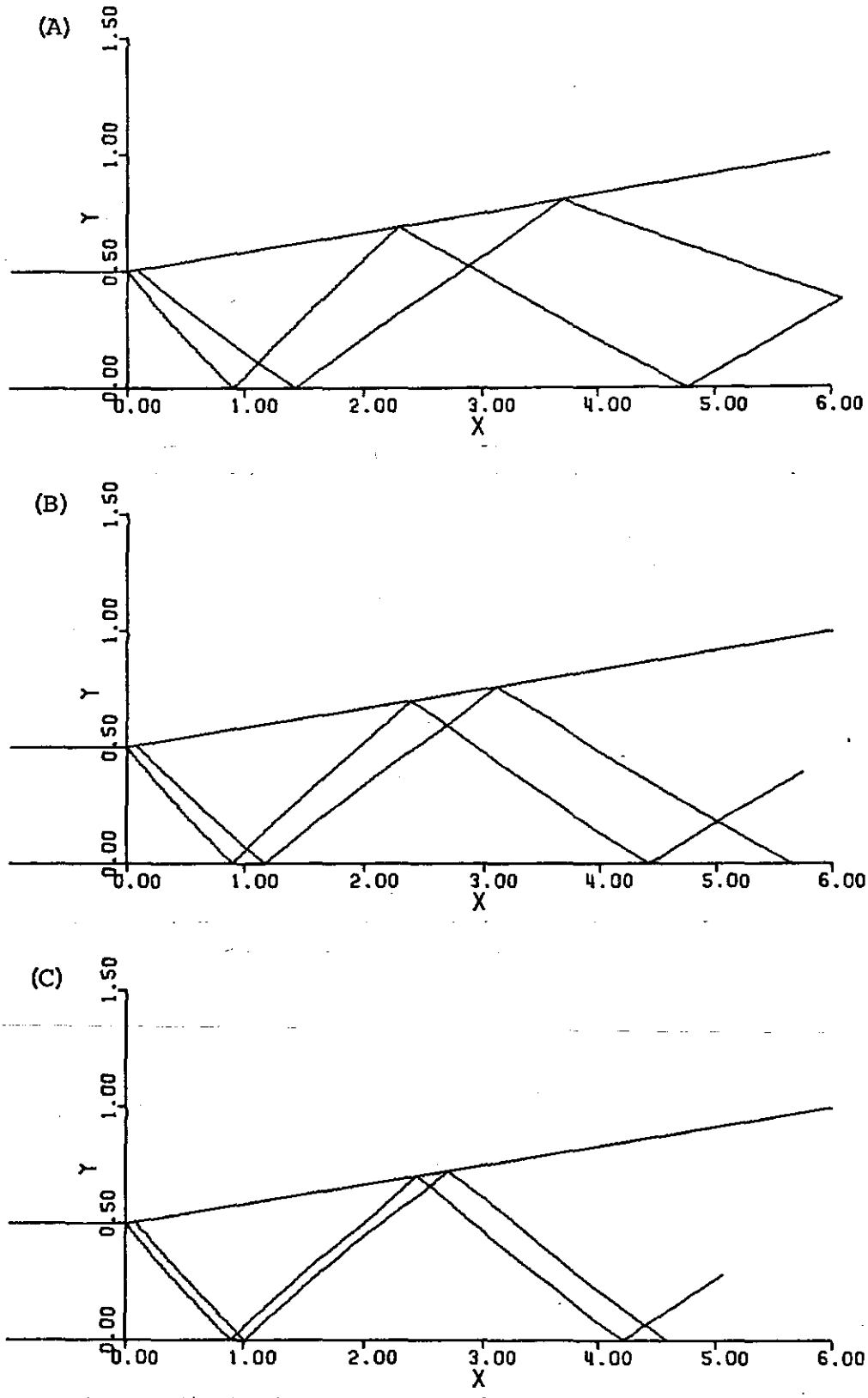


Fig. 15 Cross Wave Patterns for Supercritical Flow; Q = 0.3632 cfs, S_o = 4%: (A) q_* = 0.0 cfs/ft (B) q_* = 0.0125 cfs/ft (C) q_* = 0.0236 cfs/ft

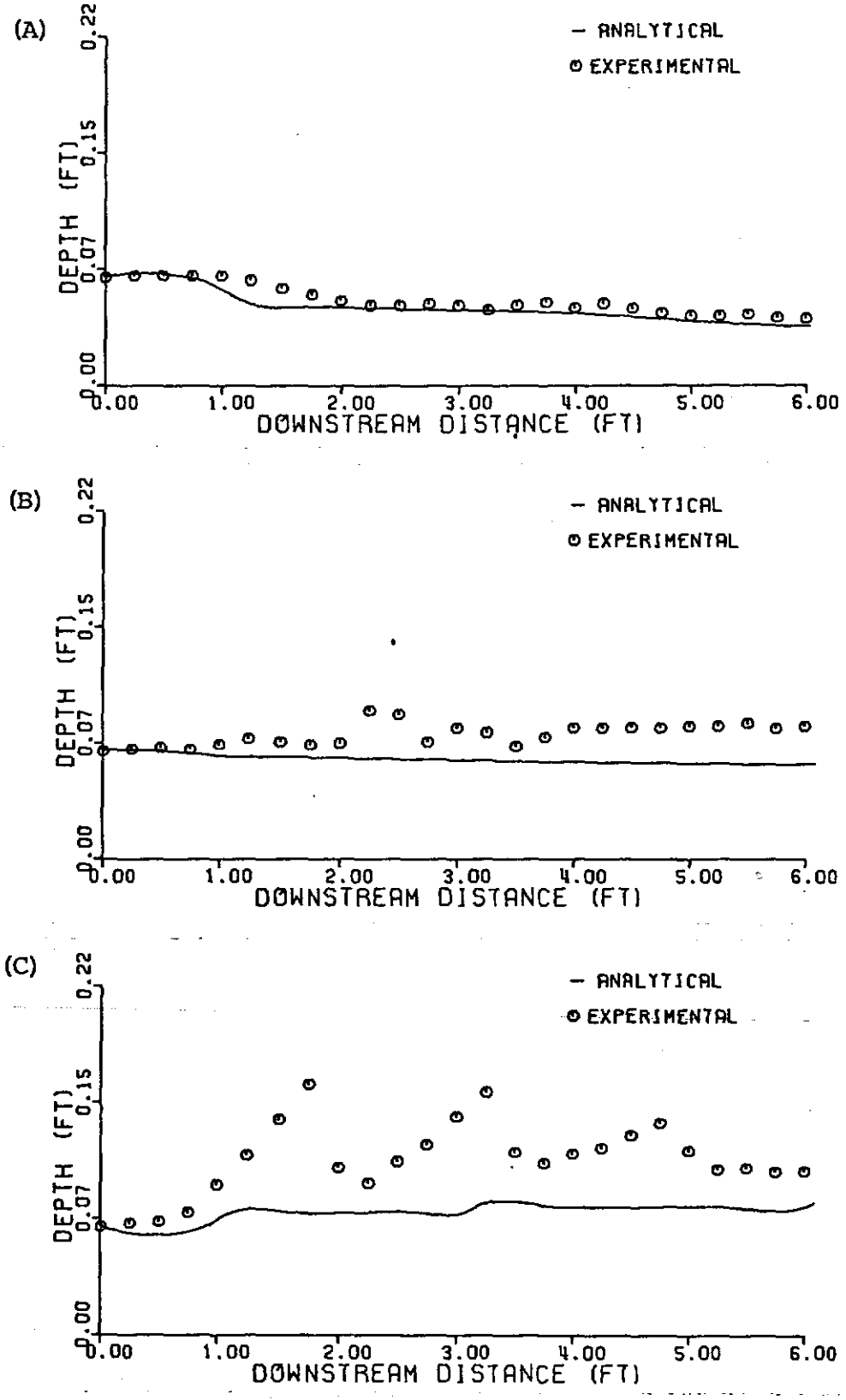


Fig. 16 Relationship Between Depth and Downstream Distance for Supercritical Flow; Q = 0.1967 cfs, S_o = 1%: (A) q_* = 0.0 cfs/ft (B) q_* = 0.0125 cfs/ft (C) q_* = 0.0236 cfs/ft

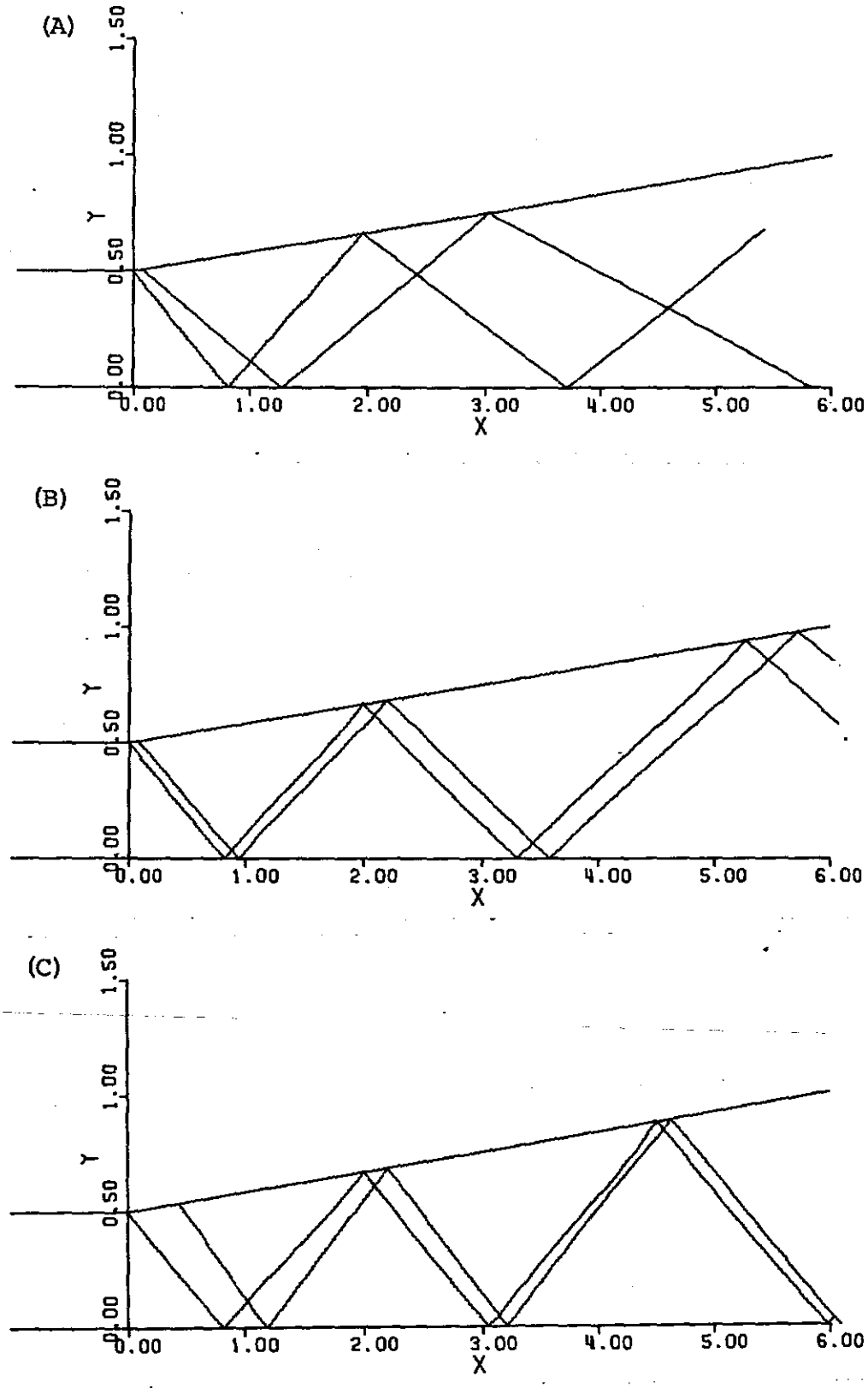


Fig. 17 Cross Wave Patterns for Supercritical Flow; Q = 0.1967 cfs, S_o = 1%: (A) q_* = 0.0 cfs/ft (B) q_* = 0.0125 cfs/ft (C) q_* = 0.0236 cfs/ft

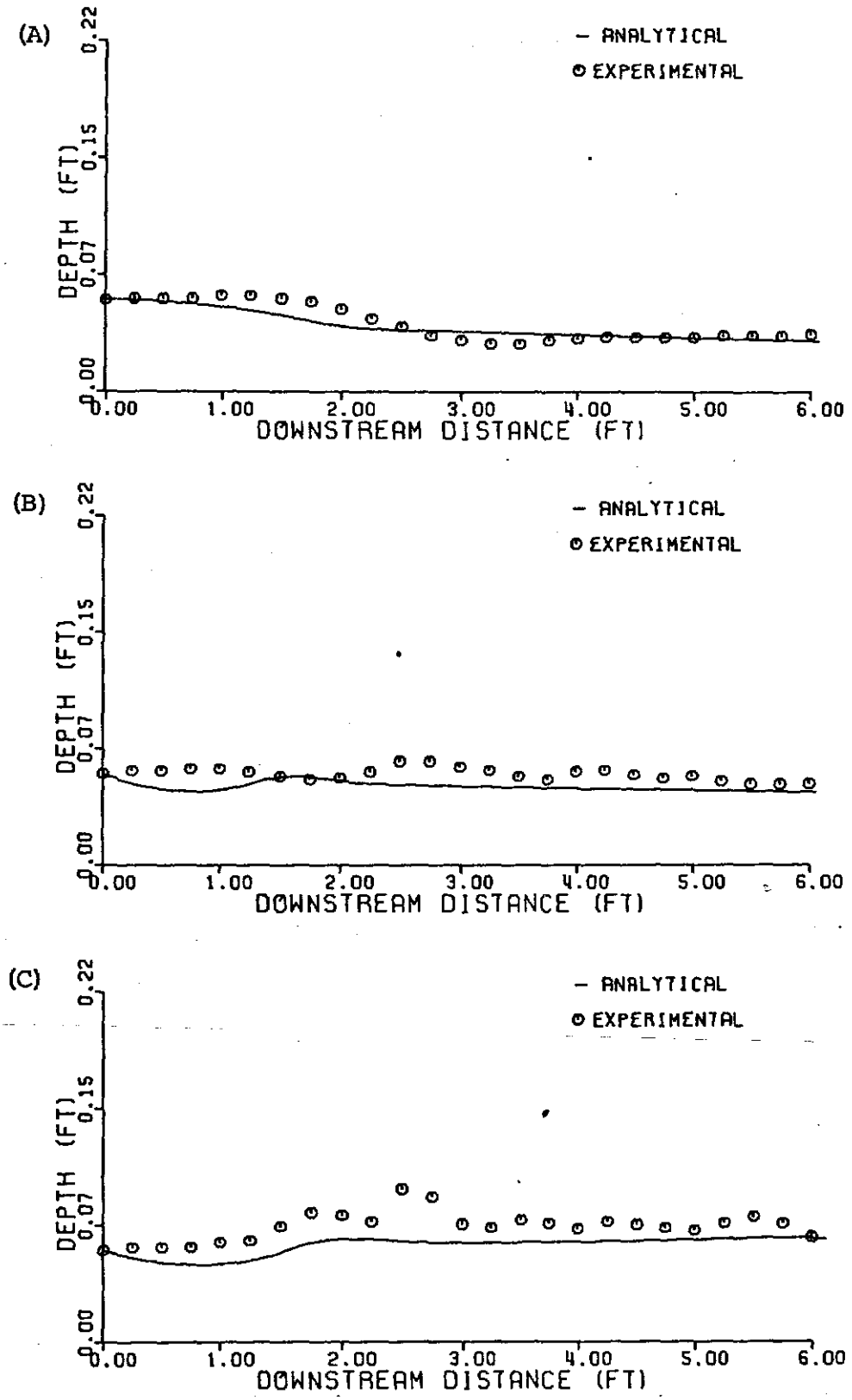
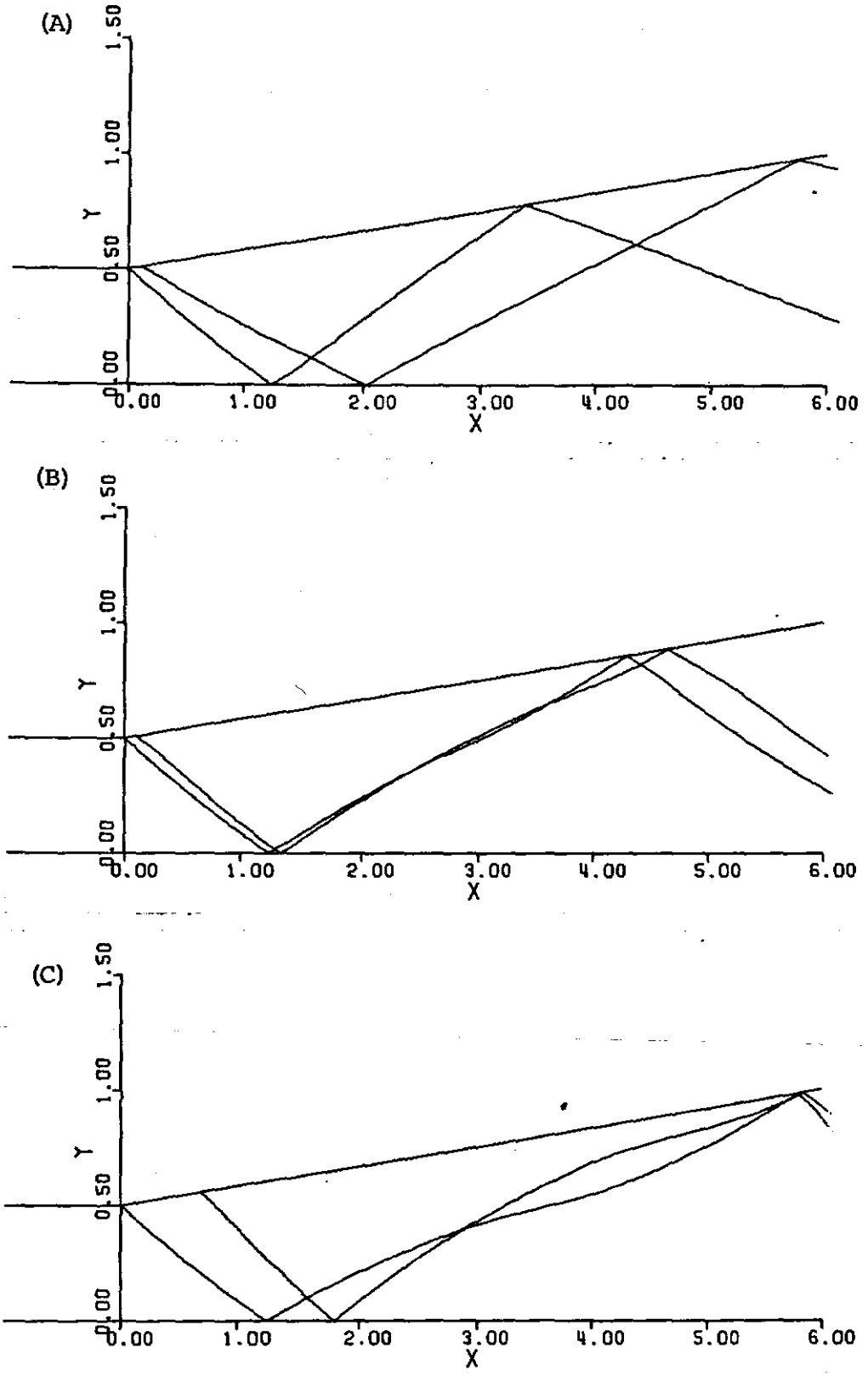


Fig. 18 Relationship Between Depth and Downstream Distance for Supercritical Flow; Q = 0.1967 cfs, S_o = 4%: (A) q_* = 0.0 cfs/ft (B) q_* = 0.0125 cfs/ft (C) q_* = 0.0236 cfs/ft



Fig, 19 Cross Wave Patterns for Supercritical Flow; Q = 0.1967 cfs, S_o = 4%: (A) q_* = 0.0 cfs/ft (B) q_* = 0.0125 cfs/ft (C) q_* = 0.0236 cfs/ft

CHAPTER VI

CONCLUSIONS

The ability to accurately predict the flow conditions for spatially varied flow in rectangular expansions is of great importance in the design or analysis of systems where this type of flow occurs. Not only is the centerline profile important, but also the sidewall profile and the general properties and conditions of the flow system.

This project was performed to establish mathematical models that can be used to determine various flow conditions. The results obtained from this study indicate that the flow conditions for both subcritical and supercritical spatially varied flow in rectangular expansions can be determined with good accuracy. This is confirmed by comparing the results computed using the developed mathematical models to that obtained from experimental analysis. This further indicates that the basic assumptions made for the analytical analysis are justified and introduce negligible error in the results.

For subcritical spatially varied flow, the following conclusions are noted:

1. The analytical and experimental centerline profiles

follow very closely for the beginning portion of the test section. The analytical profile levels off further downstream than the experimentally measured water surface profile.

2. For the same main channel flow conditions, the largest percentage increase in water depth from the point of divergence occurs when there is no lateral inflow. This is due to the fact that added lateral inflow causes an increase in downstream momentum which results in a much larger initial flow depth in the test section.

3. The discrepancies between the analytical and experimental results, demonstrated in the case when lateral inflow is added to the test section, is believed to be the result of inaccurate boundary shear calculations. In which, the effect of the lateral inflow on the shear stress computation was not taken into consideration.

For spatially varied supercritical flow through a transition, it is concluded that:

1. The characteristic method derived, based on the momentum approach, is an accurate method for analyzing such flow phenomena.

2. The analytical model developed in this study provides accurate predictions of the flow profile in the expansion section as long as the main channel flow has the dominate effect on the flow characteristics. That is to

say, if the lateral inflow is too large, the effect of the added discharge tends to cause great disturbances on the standing waves, which are used as the characteristic lines for the analytical computations. Such disturbances are believed to be responsible for the discrepancies observed between the analytical and experimental results. This is shown in Figs. 12(c) and 16(c).

3. The prevailingness of the main channel flow of the supercritical flow characteristics is determined not only by the rate of the main channel discharge, but also by the magnitude of the channel slope. It is noted in this study that larger main channel flowrates and/or channel slopes often yield better results. This can readily be seen by comparing Figs. 12(c) and 14(c); 16(c) and 18(c) for the effect of slope and Figs. 12(c) and 16(c); 14(c) and 18(c) for the effect of the rate of discharge.

4. It is also noted that the effect of a large lateral inflow is seen in the form of undular jumps along the centerline. When such phenomena occur, the characteristic method fails to predict the flow profile, since the characteristic lines, i.e. standing waves, no longer exist.

For a more complete analysis on spatially varied flow in rectangular expansions, future studies should be conducted for systems with non-linear boundaries and lateral inflow in a form other than uniform sheet flow.

APPENDIX I

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APPENDIX II

NOTATION

A = cross sectional area of a flow section, L^2;

B = width of channel, in y-direction, L;

C = Chezy friction coefficient;

d = depth of flow section, L;

f = Darcy - Weisbach friction coefficient;

F = Froude number;

g = gravitational acceleration, LT^{-2};

h_f = friction head loss, L;

K = pressure correction factor;

L = characteristic length, L;

M = momentum flux of the main flow, MLT^{-2}

M_L = momentum flux of the lateral inflow, MLT^{-2}

n = Manning roughness coefficient;

P = pressure force of the main flow, MLT^{-2}

P_L = pressure force acting perpendicular to the side-wall, MLT^{-2}

Q = main channel flowrate, $L^3\ T^{-1}$

q_* = lateral inflow per unit length, $L^2\ T^{-1}$;

R_f = boundary resistance force, M;

R = hydraulic radius, L;

S = energy grade line slope;

S_o = channel slope;

U = lateral inflow velocity, LT^{-1};

u = x - component of the main flow velocity, LT^{-1};

V = resultant main flow velocity, LT^{-1};

$\bar{V}$ = mean velocity of the main flow, LT^{-1};

v = y - component of the main flow velocity, LT^{-1};

W = force due to the weight of the fluid, MLT^{-2}

w = z - component of the main flow velocity, LT^{-1};

x = distance downstream from the origin of the divergent section, L;

α = angle of divergence;

β = momentum flux correction factor;

γ = specific weight of fluid, $ML^{-2}T^{-2}$

θ = angle the channel floor makes with the horizontal;

ρ = mass density of fluid, ML^{-3}

τ = boundary shear stress, ML^{-2};

ϕ = angle of the lateral inflow velocity vector with the flow direction.

APPENDIX III

FLOW CHARTS AND COMPUTER PROGRAMS USED IN THE ANALYSIS OF SUBCRITICAL AND SUPERCRITICAL SPATIALLY VARIED FLOW IN RECTANGULAR EXPANSIONS

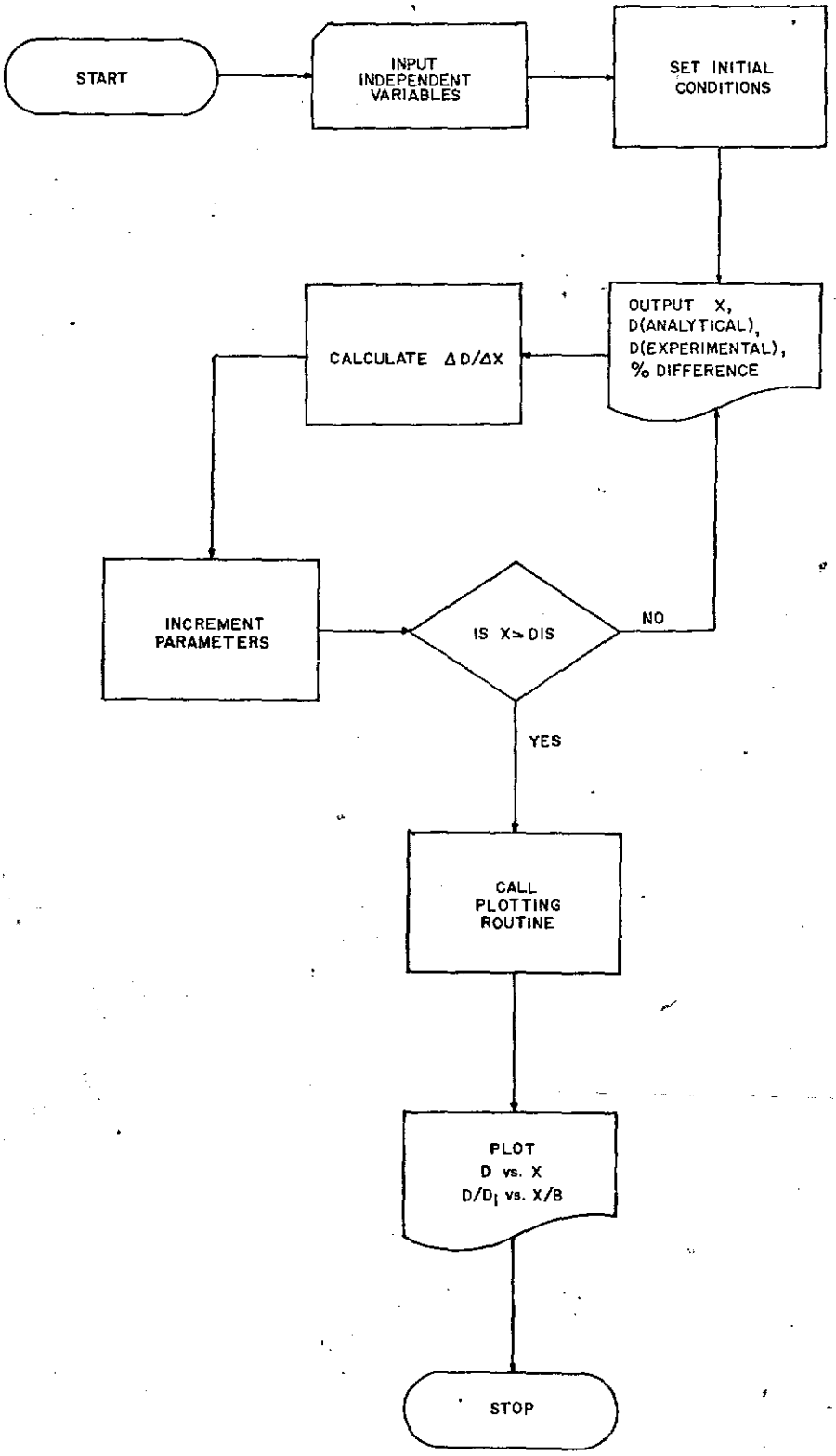


Fig. A-1 — Flow Chart for Computer Program for Subcritical Flow

Fig. A-2 COMPUTER PROGRAM FOR SUBCRITICAL FLOW

```
      C   THIS PROGRAM CALCULATES THE FLOW PROFILE FOR SPATIALLY                SUB    1
      C      VARIED SUBCRITICAL FLOW IN A RECTANGULAR EXPANSION.                 SUB    2
      C   A PLOT IS MADE COMPARING THIS PROFILE WITH THE                         SUB    3
      C      EXPERIMENTAL PROFILE.                                               SUB    4
      C                                                                          SUB    5
      C                                                                          SUB    6
      C   THE INPUT CONSISTS OF THE FOLLOWING:                                   SUB    7
      C                                                                          SUB    8
      C      L       NUMBER OF DATA SETS                                         SUB    9
      C      NEXP    NUMBER OF EXPERIMENTAL POINTS PER DATA SET                  SUB   10
      C      SO      CHANNEL SLOPE (FT/FT)                                       SUB   11
      C      BETA    MOMENTUM FLUX CORRECTION FACTOR                             SUB   12
      C      TANA    TANGENT OF THE DIVERGENT ANGLE                              SUB   13
      C      B       INITIAL WIDTH OF THE CHANNEL (FT)                           SUB   14
      C      QSTAR   LATERAL INFLOW (CFS/FT)                                     SUB   15
      C      KP      PRESSURE CORRECTION FACTOR                                  SUB   16
      C      D(1)    INITIAL DEPTH (FT)                                          SUB   17
      C      PI      RATIO OF THE CIRCUMFERENCE OF A CIRCLE                      SUB   18
      C              TO ITS DIAMETER                                             SUB   19
      C      HP      VERTICAL DISTANCE THE LATERAL INFLOW FALLS (FT)             SUB   20
      C      G       GRAVITATIONAL ACCELERATION (FT/SEC**2)                      SUB   21
      C      DX      INCREMENTAL CHANGE IN DOWNSTREAM DISTANCE (FT)              SUB   22
      C      X(1)    INITIAL DOWNSTREAM DISTANCE (FT)                            SUB   23
      C      Q       MAIN CHANNEL FLOWRATE (CFS)                                 SUB   24
      C      N       MANNING'S ROUGHNESS COFFFICIENT                             SUB   25
      C      DIS     TOTAL LENGTH OF CHANNEL REACH UNDER ANALYSIS (FT)           SUB   26
      C      GAMA    SPECIFIC WEIGHT OF THE FLUID (LB/FT**3)                     SUB   27
      C                                                                          SUB   28
      C   THE MINIMUM VALUES ON THE PLOT ARE DENOTED BY THE                     SUB   29
      C      'MIN' PREFIX AND THE SCALE FACTORS ARE DENOTED                      SUB   30
      C      BY THE 'SC' PREFIX.                                                 SUB   31
      C                                                                          SUB   32
      C                                                                          SUB   33
0001            DIMENSION DATA(1024)                                           SUB   34
0002            REAL A(10),D(125),DP(125),X(125),XP(125),DDI(125),DDIP(125),   SUB   35
               (XB(125),XBP(125),KP,EXX(25),EXD(25),EXXB(25),EXDDI(25),NUM,N,  SUB   36
               (MINXP,MINXBP,MINDP,MINDD,MINEX,MINED,MINEXB,MINEXD             SUB   37
C003            INTEGER H                                                      SUB   38
0004            CALL PLOTS(DATA,4096)                                          SUB   39
0005            CALL FACTOR (0.7)                                              SUB   40
0006            READ(5,1050) L,NEXP                                            SUB   41
0007            READ(5,1060)MINXP,MINDP,MINXBP,MINDD,MINEX,MINED,MINEXB,MINEXD SUB   42
0008            READ(5,1060)SCXP,SCDP,SCXBP,SCDD,SCEX,SCED,SCEXB,SCEXD         SUB   43
0009            DO 120 II=1,L                                                  SUB   44
      C                                                                        SUB   45
      C   INITIALIZING THE POSITION OF THE PLOTTER PEN                         SUB   46
      C                                                                        SUB   47
0010            CALL PLOT(0.0,-11.0,-3)                                        SUB   48
0011            CALL PLOT(0.0,3.25,-3)                                         SUB   49
0012            READ(5,1060) SO,BETA,TANA,B,QSTAR,KP,D(1),PI                   SUB   50
0013            READ(5,1060)HP,G,DX,X(1),Q,N,DIS,GAMA                          SUB   51
0014            SOP=SO*100.                                                    SUB   52
0015            BO=B                                                           SUB   53
C016            ALFA=ATAN(TANA)                                                SUB   54
0017            ALPHA=ALFA*180./PI                                             SUB   55
0018            WRITE(6,1000) Q                                                SUB   56
0019            WRITE(6,1010) QSTAR                                            SUB   57
0020            WRITE(6,1020)                                                  SUB   58
```

```
0021              WRITE(6,1030)                                          SUB   59
0022              DI=D(1)                                                SUB   60
                C                                                        SUB   61
                C   ACCUMULATION OF THE EXPERIMENTAL DATA                SUB   62
                C                                                        SUB   63
0023              DO 10 JJ=1,NEXP                                        SUB   64
0024              READ(5,1070)EXX(JJ),EXD(JJ)                            SUB   65
0025              EXXB(JJ)=EXX(JJ)/BO                                    SUB   66
0026           10 EXDDI(JJ)=EXD(JJ)/DI                                   SUB   67
0027              THETA=ATAN(SO)                                         SUB   68
0028              PHI=(PI/2.)-THETA                                      SUB   69
0029              J=1                                                    SUB   70
0030              JJ=0                                                   SUB   71
0031           20 DDI(J)=D(J)/DI                                         SUB   72
0032              XB(J)=X(J)/BO                                          SUB   73
0033              IF(((J-6)/5)*5.NE.(J-6)) GO TO 30                      SUB   74
0034              JJ=JJ+1                                                SUB   75
0035              PDIFF=ABS(D(J)-EXD(JJ))*100./D(J)                      SUB   76
0036              WRITE(6,1040) X(J),D(J),EXD(JJ),PDIFF                  SUB   77
0037           30 V=Q/(D(J)*B)                                           SUB   78
0038              U=SQRT(2.*G*HP)                                        SUB   79
0039              AREA=B*D(J)                                            SUB   80
0040              PERI=B+2.*D(J)                                         SUB   81
0041              RAD=AREA/PERI                                          SUB   82
0042              TAU=(0.45*GAMA*V*V*N*N)/(RAD**0.3333)                  SUB   83
                C                                                        SUB   84
                C   A(1) THROUGH A(8) ARE THE INDIVIDUAL TERMS IN THE    SUB   85
                C      EXPRESSION FOR DD/DX                              SUB   86
                C                                                        SUB   87
0043              A(1)=SO                                                SUB   88
0044              A(2)=(QSTAR*U*COS(PHI)*COS(ALFA))/(G*D(J)*B)           SUB   89
0045              A(3)=((2.)*BETA*V*V*TANA)/(G*B)                        SUB   90
0046              A(4)=((-2.)*BETA*V*QSTAR*COS(PHI)*COS(ALFA))/(G*B*D(J))SUB   91
0047              A(5)=(-1.)*TAU/(GAMA*D(J))-2.*TAU/(GAMA*B)             SUB   92
0048              A(6)=2.*(KP*D(J)/B)*COS(THETA)*(SIN(ALFA)-TAN(ALFA))   SUB   93
0049              A(7)=2.*KP*COS(THETA)                                  SUB   94
0050              A(8)=((-1.)*BETA*V*V)/(G*D(J))                         SUB   95
0051              NUM=0.0                                                SUB   96
                C                                                        SUB   97
                C   CALCULATION OF DD/DX                                 SUB   98
                C                                                        SUB   99
0052              DO 40 I=1,8                                            SUB  100
0053           40 NUM=NUM+A(I)                                           SUB  101
0054              DEN=A(7)+A(8)                                          SUB  102
0055              DDDX=NUM/DEN                                           SUB  103
0056              J=J+1                                                  SUB  104
0057              H=J-1                                                  SUB  105
0058              D(J)=D(H)+DDDX*DX                                      SUB  106
0059              X(J)=X(H)+DX                                           SUB  107
0060              IF(X(J).GE.DIS) GO TO 50                               SUB  108
0061              B=B+2.*DX*TANA                                         SUB  109
0062              Q=Q+QSTAR*DX                                           SUB  110
0063              GO TO 20                                               SUB  111
                C                                                        SUB  112
                C   EVERY FIFTH ANALYTICAL POINT IS PLOTTED             SUB  113
                C                                                        SUB  114
0064           50 K=(J/5)+1                                              SUB  115
0065              DO 60 I=2,K                                            SUB  116
```

```
0066              XP(I)=X((I-1)*5+1)                                          SUB  117
0067              DP(I)=D((I-1)*5+1)                                          SUB  118
0068              XBP(I)=XB((I-1)*5+1)                                        SUB  119
0069           60 DDIP(I)=DDI((I-1)*5+1)                                      SUB  120
0070              XP(1)=X(1)                                                  SUB  121
0071              DP(1)=D(1)                                                  SUB  122
0072              XBP(1)=XB(1)                                                SUB  123
0073              DDIP(1)=DDI(1)                                              SUB  124
                C                                                             SUB  125
                C   THE SUBSCRIPT (K+1) IS A MINIMUM FOR THE PARTICULAR       SUB  126
                C      VARIABLE TO BE PLOTTED                                 SUB  127
                C                                                             SUB  128
0074              XP(K+1)=MINXP                                               SUB  129
0075              DP(K+1)=MINDP                                               SUB  130
0076              XBP(K+1)=MINXBP                                             SUB  131
0077              DDIP(K+1)=MINDD                                             SUB  132
0078              EXX(K+1)=MINEX                                              SUB  133
0079              EXD(K+1)=MINED                                              SUB  134
0080              EXXB(K+1)=MINEXB                                            SUB  135
0081              EXDDI(K+1)=MINEXD                                           SUB  136
                C                                                             SUB  137
                C   THE SUBSCRIPT (K+2) IS THE VALUE OF ONE DIVISION         SUB  138
                C      ON THE AXIS OF THE PLOT                                SUB  139
                C                                                             SUB  140
0082              XP(K+2)=SCXP                                                SUB  141
0083              DP(K+2)=SCDP                                                SUB  142
0084              XBP(K+2)=SCXBP                                              SUB  143
0085              DDIP(K+2)=SCDD                                              SUB  144
0086              EXX(K+2)=SCEX                                               SUB  145
0087              EXD(K+2)=SCED                                               SUB  146
0088              EXXB(K+2)=SCEXB                                             SUB  147
0089              EXDDI(K+2)=SCEXD                                            SUB  148
                C                                                             SUB  149
                C   SCALING OF THE VARIABLES TO BE PLOTTED                    SUB  150
                C                                                             SUB  151
0090              DO 70 I=1,K                                                 SUB  152
0091              XP(I)=(XP(I)-XP(K+1))/XP(K+2)                               SUB  153
0092              DP(I)=(DP(I)-DP(K+1))/DP(K+2)                               SUB  154
0093              XBP(I)=(XBP(I)-XBP(K+1))/XBP(K+2)                           SUB  155
0094              DDIP(I)=(DDIP(I)-DDIP(K+1))/DDIP(K+2)                       SUB  156
0095              EXX(I)=(EXX(I)-EXX(K+1))/EXX(K+2)                           SUB  157
0096              EXD(I)=(EXD(I)-EXD(K+1))/EXD(K+2)                           SUB  158
0097              EXXB(I)=(EXXB(I)-EXXB(K+1))/EXXB(K+2)                       SUB  159
0098           70 EXDDI(I)=(EXDDI(I)-EXDDI(K+1))/EXDDI(K+2)                   SUB  160
                C                                                             SUB  161
                C   LOCATING AND LABELLING THE AXES                          SUB  162
                C                                                             SUB  163
0099              CALL AXIS(0.0,0.0,'DOWNSTREAM DISTANCE (FT)',-24,6.,0.0,XP(K+1),  SUB  164
                 1XP(K+2))                                                    SUB  165
0100              CALL AXIS(0.0,0.0,'DEPTH (FT)',10,3.,90.,DP(K+1),DP(K+2))   SUB  166
0101              CALL AXIS(0.0,5.0,'DOWNSTREAM DISTANCE/INITIAL WIDTH',-33,6.,0.0,  SUB  167
                 (XBP(K+1),XBP(K+2))                                          SUB  168
0102              CALL AXIS(0.0,5.0,'DEPTH/INITIAL DEPTH',19,3.,90.,DDIP(K+1),DDIP(  SUB  169
                 (K+2))                                                       SUB  170
                C                                                             SUB  171
                C   SUBROUTINE SMOOTH IS USED FOR THE ANALYTICAL PROFILE      SUB  172
                C                                                             SUB  173
0103              CALL SMOOTH(XP(1),DP(1),0)                                  SUB  174
```

```
0104              KK=K-1                                                        SUB  175
0105              DO 80 I=2,KK                                                  SUB  176
0106           80 CALL SMOOTH(XP(I),DP(I),-2)                                   SUB  177
0107              CALL SMOOTH(XP(K),DP(K),-999)                                 SUB  178
0108              CALL PLOT(0.0,0.0,-3)                                         SUB  179
                C                                                               SUB  180
                C   SUBROUTINE SYMBOL IS USED FOR THE EXPERIMENTAL RESULTS     SUB  181
                C                                                               SUB  182
0109              DO 90 JJ=1,NEXP                                               SUB  183
0110           90 CALL SYMBOL(EXX(JJ),EXD(JJ),.08,1,0.0,-1)                     SUB  184
0111              CALL SYMBOL(1.0,3.0,0.11,'_ ANALYTICAL',0.0,12)               SUB  185
0112              CALL SYMBOL(1.0,2.75,.157,'  EXPERIMENTAL ',0.0,15)           SUB  186
0113              CALL PLOT(0.0,5.0,-3)                                         SUB  187
0114              CALL SMOOTH(XBP(1),DDIP(1),0)                                 SUB  188
0115              DO 100 I=2,KK                                                 SUB  189
0116          100 CALL SMOOTH(XBP(I),DDIP(I),-2)                                SUB  190
0117              CALL SMOOTH(XBP(K),DDIP(K),-999)                              SUB  191
0118              CALL PLOT(0.0,0.0,-3)                                         SUB  192
0119              DO 110 JJ=1,NEXP                                              SUB  193
0120          110 CALL SYMBOL(EXXB(JJ),EXDDI(JJ),.08,1,0.0,-1)                  SUB  194
0121              CALL SYMBOL(1.0,3.0,0.11,'_ ANALYTICAL',0.0,12)               SUB  195
0122              CALL SYMBOL(1.0,2.75,.157,'  EXPERIMENTAL ',0.0,15)           SUB  196
0123         1000 FORMAT('1',////////,38X,'MAIN CHANNEL FLOWRATE:',F6.3,' CFS'/) SUB  197
0124         1010 FORMAT(' ',37X,'LATERAL INFLOW:',F7.4,' CFS/FT')              SUB  198
0125         1020 FORMAT(' ',37X,'DOWNSTREAM',14X,'DEPTH (FT)',13X,'PERCENT')   SUB  199
0126         1030 FORMAT(' ',37X,'DISTANCE (FT)',4X,'ANALYTICAL',2X,'EXPERIMENTAL', SUB  200
                    (4X,'DIFFERENCE')                                           SUB  201
0127         1040 FORMAT(' ',40X,F5.2,10X,F7.4,7X,F6.3,9X,F6.3)                 SUB  202
0128         1050 FORMAT(2I3)                                                   SUB  203
0129         1060 FORMAT(8F10.5)                                                SUB  204
0130         1070 FORMAT(2F10.2)                                                SUB  205
                C                                                               SUB  206
                C   SHIFTS PLOTTER PEN FOR ANOTHER SET OF DATA                  SUB  207
                C                                                               SUB  208
0131          120 CALL PLOT(12.0,0.0,-3)                                        SUB  209
0132              CALL PLOT(0.0,0.0,999)                                        SUB  210
0133              RETURN                                                        SUB  211
0134              END                                                           SUB  212
```

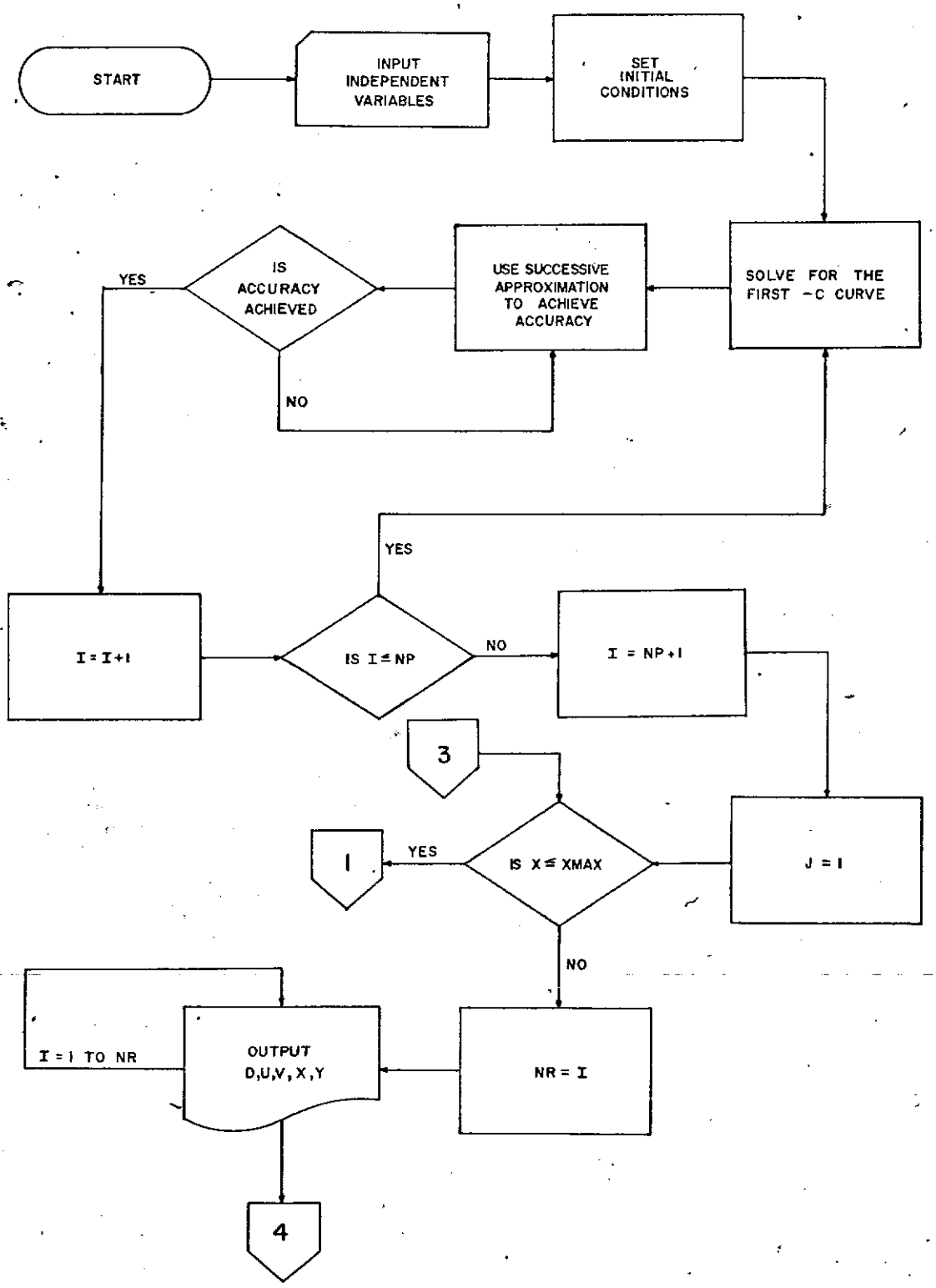


Fig. A-3a – Flow Chart for Computer Program for Supercritical Flow

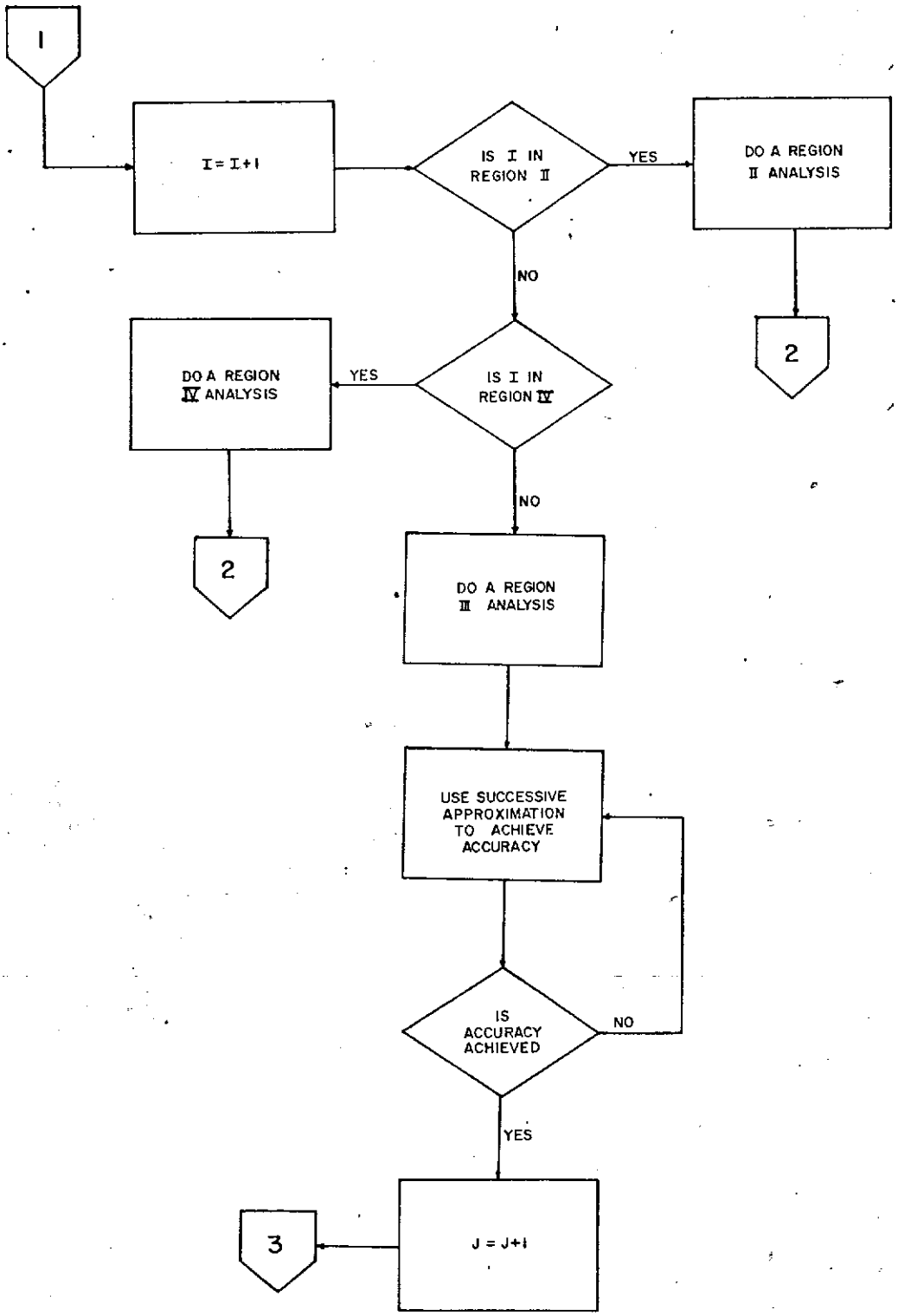
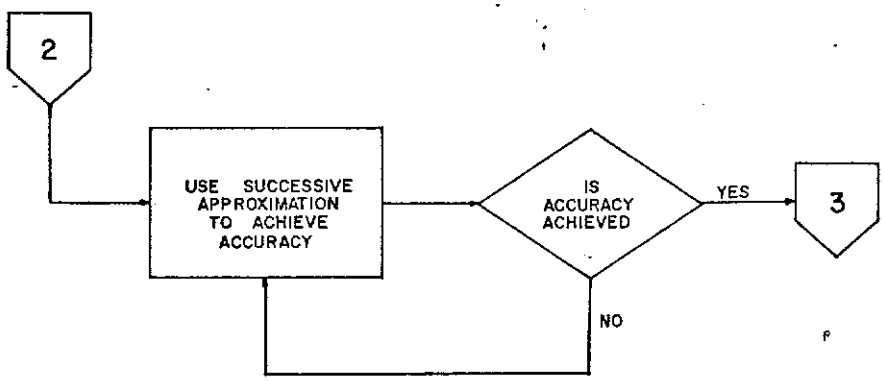


Fig. A-3b — Flow Chart for Computer Program for Supercritical Flow



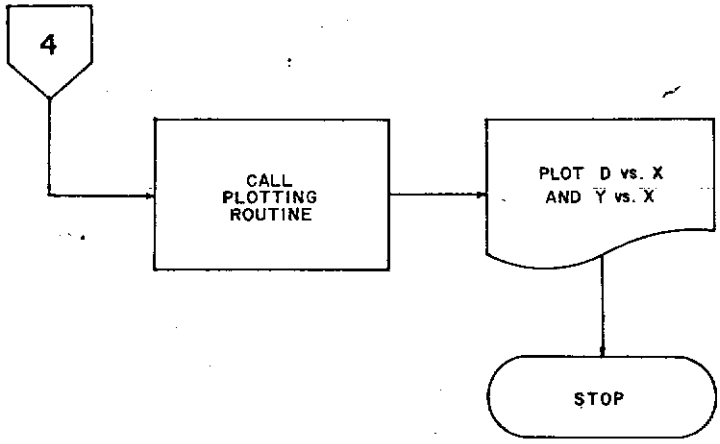


Fig. A-3c – Flow Chart for Computer Program for Supercritical Flow

Fig. A-4 COMPUTER PROGRAM FOR SUPERCRITICAL FLOW

```
      C                                                                      SUP    1
      C     THIS PROGRAM CALCULATES THE FLOW PROFILE AND THE CROSS WAVE      SUP    2
      C        PATTERNS FOR SPATIALLY VARIED SUPERCRITICAL FLOW IN A REC-    SUP    3
      C        TANGULAR EXPANSION.  THE METHOD OF CHARACTERISTICS IS USED    SUP    4
      C        TO DETERMINE THE FLOW CONDITIONS AT EACH POINT IN THE FLOW    SUP    5
      C        REGIME.  PLOTS ARE MADE OF THE CROSS WAVE PATTERNS; AND THE   SUP    6
      C        ANALYTICAL AND EXPERIMENTAL CENTERLINE PROFILES.              SUP    7
      C                                                                      SUP    8
      C     THE INPUT CONSISTS OF THE FOLLOWING:                             SUP    9
      C                                                                      SUP   10
      C        UO      THE INITIAL VELOCITY IN THE X-DIRECTION (FPS)         SUP   11
      C        DO      THE INITIAL DEPTH AT POINT 0 (FT)                     SUP   12
      C        Q       THE MAIN CHANNEL FLOWRATE (CFS)                       SUP   13
      C        THETA   BOTTOM SLOPE OF THE CHANNEL                           SUP   14
      C        ALPHA   ANGLE OF THE SIDEWALL WITH THE CENTERLINE             SUP   15
      C        B       WIDTH OF THE INITIAL CHANNEL   (FT)                   SUP   16
      C        ACC     DESIRED ACCURACY FOR THE CALCULATION OF U(I)          SUP   17
      C        DELYO   THE CHANGE IN Y-DISTANCE FROM POINT 0 TO POINT 1      SUP   18
      C        QSTAR   LATERAL INFLOW (CFS/FT)                               SUP   19
      C        NP      THE NUMBER OF POINTS IN THE FIRST -C CURVE            SUP   20
      C        XMAX    THE MAXIMUM LENGTH ALONG CENTERLINE FOR WHICH ANALYSIS SUP   21
      C                IS DESIRED                                            SUP   22
      C        IDATA   THE NUMBER OF DATA SETS                               SUP   23
      C        NEXP    THE NUMBER OF EXPERIMENTAL POINTS                     SUP   24
      C                                                                      SUP   25
      C     THE MINIMUM VALUES ON THE PLOTS ARE DENOTED BY THE 'MIN' PREFIX  SUP   26
      C        AND THE SCALE FACTORS ARE DENOTED BY THE 'SC' PREFIX.         SUP   27
      C                                                                      SUP   28
      C     NOTE: ALPHA AND THETA ARE GIVEN IN DEGREES                       SUP   29
      C                                                                      SUP   30
      C                                                                      SUP   31
0001          DIMENSION DATA (1024)                                          SUP   32
0002          CALL PLOTS (DATA,4096)                                         SUP   33
0003          CALL FACTOR(0.7)                                               SUP   34
0004          REAL XPT(500),YPT(500),XP(500),DP(500),EXXP(100),EXDP(100),    SUP   35
             (MINXT,MINXP,MINDP,MINYT                                        SUP   36
0005          COMMON U(500),D(500),V(500),X(500),Y(500),I,NP,NPP,J,THETA,    SUP   37
             (ALPHA,UMM,UNP,VMM,VNP,UMM,DNP,DELUN,DELUP,DELVN,DELVP,DELXN,   SUP   38
             (DELXP,DELYN,DELYP,DELX,DELYO,GRAV,DELU,UO,QSTAR,YO,XU,FNP,QI,UO SUP   39
0006          READ (5,1000) IDATA                                            SUP   40
0007          READ(5,1010) MINXT,MINXP,MINDP,MINYT,SCXPT,SCXP,SCDP,SCYPT     SUP   41
0008          DO 380 ID=1,IDATA                                              SUP   42
      C                                                                      SUP   43
      C        INITIALIZING THE POSITION OF THE PLOTTER PEN                  SUP   44
      C                                                                      SUP   45
0009          CALL PLOT(5.0,-11.0,-3)                                        SUP   46
0010          CALL PLOT(5.0,3.25,-3)                                         SUP   47
0011          READ(5,1000) NEXP                                              SUP   48
0012          READ(5,1020)UO,DO,THETA,ALPHA,B,ACC,Q,QSTAR                    SUP   49
0013          READ(5,1030)NP,XMAX,DELYO                                      SUP   50
0014          THETA=THETA*6.2832/360.0                                       SUP   51
0015          ALPHA=ALPHA*6.2832/360.0                                       SUP   52
0016          SLOPE=100.*TAN(THETA)                                          SUP   53
      C                                                                      SUP   54
      C        ACCUMULATION OF THE EXPERIMENTAL POINTS                       SUP   55
      C                                                                      SUP   56
0017          DO 10 II=1,NEXP                                                SUP   57
0018       10 READ(5,1040) EXXP(II),EXDP(II)                                 SUP   58
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```
0019              NPP=NP+1                                                    SUP   59
0020              GRAV=32.174                                                 SUP   60
0021              YO=B/2.                                                     SUP   61
0022              XO=0.0                                                      SUP   62
0023              VO=0.0                                                      SUP   63
0024           20 DELYO=DELYO+0.1*DELYO                                       SUP   64
0025              J=0                                                         SUP   65
0026              DELY=-(YO+DELYO)/(NP-1)                                     SUP   66
                C                                                             SUP   67
                C    THE FOLLOWING EQUATIONS ARE FOR A TYPE 1 ANALYSIS.  THIS TYPE   SUP   68
                C       OF ANALYSIS IS  FOR THE FIRST NP POINTS (THE POINTS ALONG THE SUP   69
                C       FIRST PC CURVE).                                      SUP   70
                C                                                             SUP   71
0027              DO 40 I=1,NP                                                SUP   72
0028              V(I)=0.0                                                    SUP   73
0029              UX=100.                                                     SUP   74
0030              UMM=UO                                                      SUP   75
0031              IF(I.NE.1) UMM=U(I-1)                                       SUP   76
0032              DMM=DO                                                      SUP   77
0033              IF(I.NE.1) DMM=D(I-1)                                       SUP   78
0034              IF(I.NE.1) DELYO=DELY                                       SUP   79
0035           30 DELU=-UMM*TAN(THETA)*DELYO/DMM/SQRT(UMM**2/GRAV/DMM-1.)     SUP   80
0036              U(1)=UO+DELU                                                SUP   81
0037              IF(I.NE.1) U(I)=U(I-1)+DELU                                 SUP   82
0038              Y(I)=YO+DELYO                                               SUP   83
0039              IF(I.NE.1) Y(I)=Y(I-1)+DELY                                 SUP   84
0040              DELX=DELYO*(UMM*GRAV-(UMM**2))/DMM/GRAV/SQRT(UMM**2/GRAV/DMM-1.)  SUP   85
0041              X(I)=XO+DELX                                                SUP   86
0042              IF(I.NE.1) X(I)=X(I-1)+DELX                                 SUP   87
0043              IF(I.NE.1) DELXN=X(I)-X(I-1)                                SUP   88
0044              IF(I.NE.1) DELYN=Y(I)-Y(I-1)                                SUP   89
0045              CALL DEPTH                                                  SUP   90
0046              IF(ABS(UX-U(I)).LE.ACC) GO TO 40                            SUP   91
0047              UMM=(U(I)+UO)/2.0                                           SUP   92
0048              IF(I.NE.1) UMM=(U(I)+U(I-1))/2.                             SUP   93
0049              DMM=(D(I)+DO)/2.0                                           SUP   94
0050              IF(I.NE.1) DMM=(D(I)+D(I-1))/2.                             SUP   95
0051              UX=U(I)                                                     SUP   96
0052              GO TO 30                                                    SUP   97
0053           40 CONTINUE                                                    SUP   98
0054              I=NP+1                                                      SUP   99
0055              J=1                                                         SUP  100
0056           50 CONTINUE                                                    SUP  101
0057              JN=J*(NP+1)                                                 SUP  102
0058              IF(I.EQ.JN)GOTO90                                           SUP  103
0059              JNP=(J+1)*NP+J                                              SUP  104
0060              IF(I.EQ.JNP)GOTO120                                         SUP  105
                C                                                             SUP  106
                C    THESE EQUATIONS ARE FOR A REGION 4 ANALYSIS. THIS TYPE OF ANALYSIS SUP  107
                C       IS FOR THE POINTS IN THE INTERIOR REGION OF FLOW.      SUP  108
                C                                                             SUP  109
0061           60 UMM=U(I-1)                                                  SUP  110
0062              UNP=U(I-NP)                                                 SUP  111
0063              VMM=V(I-1)                                                  SUP  112
0064              VNP=V(I-NP)                                                 SUP  113
0065              DMM=D(I-1)                                                  SUP  114
0066              DNP=D(I-NP)                                                 SUP  115
0067              UX=100.                                                     SUP  116
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0068          70 FNP=SQRT((UNP**2+VNP**2)/GRAV/DNP)                                  SUP  117
0069             FMM=SQRT((UMM**2+VMM**2)/GRAV/DMM)                                  SUP  118
0070             GM=(-UMM*VMM+DMM*GRAV*SQRT(FMM**2-1.))/(DMM*GRAV-UMM**2)            SUP  119
0071             GP=(-UNP*VNP-DNP*GRAV*SQRT(FNP**2-1.))/(DNP*GRAV-UNP**2)            SUP  120
0072             X(I)=(GM*X(I-1)-GP*X(I-NP)-Y(I-1)+Y(I-NP))/(GM-GP)                  SUP  121
0073             Y(I)=GM*(X(I)-X(I-1))+Y(I-1)                                        SUP  122
0074             G1=U(I-NP)-U(I-1)                                                   SUP  123
0075             G2=V(I-NP)-V(I-1)                                                   SUP  124
0076             G4=(UMM*VMM/GRAV-DMM*SQRT(FMM**2-1.))/(DMM-VMM**2/GRAV)             SUP  125
0077             G6=(DNP*SQRT(FNP**2-1.)+UNP*VNP/GRAV)/(DNP-VNP**2/GRAV)             SUP  126
0078             G7=-UMM*TAN(THETA)*(Y(I)-Y(I-1))/(DMM-VMM**2/GRAV)                  SUP  127
0079             G8=-UNP*TAN(THETA)*(Y(I)-Y(I-NP))/(DNP-VNP**2/GRAV)                 SUP  128
0080             DELUP=(G8*G2-G7-G1*G4)/(G4-G6)                                      SUP  129
0081             DELUN=(G2-G7*G8-G6*G1)/(G4-G6)                                      SUP  130
0082             DELVP=G8+DELUP*G6                                                   SUP  131
0083             DELVN=G7+G4*DELUN                                                   SUP  132
0084             U1=U(I-NP)+DELUP                                                    SUP  133
0085             U2=U(I-1)+DELUN                                                     SUP  134
0086             V1=V(I-NP)+DELVP                                                    SUP  135
0087             V2=V(I-1)+DELVN                                                     SUP  136
0088             U(I)=(U1+U2)/2.0                                                    SUP  137
0089             V(I)=(V1+V2)/2.0                                                    SUP  138
0090             DELXP=X(I)-X(I-NP)                                                  SUP  139
0091             DELXN=X(I)-X(I-1)                                                   SUP  140
0092             DELYP=Y(I)-Y(I-NP)                                                  SUP  141
0093             DELYN=Y(I)-Y(I-1)                                                   SUP  142
0094             CALL DEPTH                                                          SUP  143
0095             IF(ABS(UX-U(I)).LE.ACC) GO TO 80                                    SUP  144
0096             UX=U(I)                                                             SUP  145
0097             UMM=(U(I-1)+U(I))/2.                                                SUP  146
0098             UNP=(U(I-NP)+U(I))/2.                                               SUP  147
0099             VMM=(V(I-1)+V(I))/2.0                                               SUP  148
0100             VNP=(V(I-NP)+V(I))/2.0                                              SUP  149
0101             DMM=(D(I-1)+D(I))/2.0                                               SUP  150
0102             DNP=(D(I-NP)+D(I))/2.0                                              SUP  151
0103             GOTO70                                                              SUP  152
0104          80 I=I+1                                                               SUP  153
0105             GOTO50                                                              SUP  154
               C                                                                     SUP  155
               C    THE FOLLOWING EQUATIONS ARE FOR A REGION 2 ANALYSIS.  THIS TYPE OF SUP  156
               C        ANALYSIS IS FOR THE POINTS ALONG THE SIDEWALL.  THE NUMBER FOR SUP  157
               C        THESE POINTS CORRESPONDS TO THE EQUATION: I=J*(NP+1), WHERE J  SUP  158
               C        IS THE NUMBER OF THE -C CURVE IN WHICH THIS POINT IS FOUND.    SUP  159
               C                                                                     SUP  160
0106          90 UNP=U(I-NP)                                                        SUP  161
0107             VNP=V(I-NP)                                                         SUP  162
0108             DNP=D(I-NP)                                                         SUP  163
0109             UX=100.                                                             SUP  164
0110         100 FNP=SQRT((UNP**2+VNP**2)/GRAV/DNP)                                  SUP  165
0111             CALL DYDX                                                           SUP  166
0112             IF(DELX.EQ.1000.) GO TO 370                                         SUP  167
0113             DELXP=X(I)-X(I-NP)                                                  SUP  168
0114             DELYP=Y(I)-Y(I-NP)                                                  SUP  169
0115             C2=CSTAR*ABS(X(I)-X(I-NP))/COS(ALPHA)                               SUP  170
0116             PHIN=(U1+U2)/ABS(DELXP)/ABS(DELYP)-UNP*TAN(THETA)+(UNP**2+VNP       SUP  171
                (**2)*.2/DNP/GRAV/ABS(DELXP)/ABS(DELYP)                             SUP  172
0117             UTOP=PHIN*DELYP-(DNP-VNP**2/GRAV)*(U(I-NP)*TAN(ALPHA)-V(I-NP))     SUP  173
0118             UBOT=DELYP*(DNP-UNP**2/GRAV-U1*UNP/2./DNP/GRAV/ABS(DELYP))/        SUP  174
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                (DELXP*TAN(ALPHA)*(DNP-VNP**2/GRAV)                                SUP  175
0119            DELU=UTOP/UBOT                                                     SUP  176
0120            DELV=(U(I-NP)+DELU)*TAN(ALPHA)-V(I-NP)                             SUP  177
0121            U(I)=U(I-NP)+DELU                                                  SUP  178
0122            V(I)=V(I-NP)+DELV                                                  SUP  179
0123            DELUP=U(I)-U(I-NP)                                                 SUP  180
0124            DELVP=V(I)-V(I-NP)                                                 SUP  181
0125            CALL DEPTH                                                         SUP  182
0126            IF(ABS(D(I)-DX).LE.ACC) GO TO 110                                  SUP  183
0127            DX=D(I)                                                            SUP  184
0128            UNP=(U(I)+U(I-NP))/2.0                                             SUP  185
0129            VNP=(V(I)+V(I-NP))/2.0                                             SUP  186
0130            DNP=(D(I)+D(I-NP))/2.0                                             SUP  187
0131            GO TO 100                                                          SUP  188
0132          110 IF(X(I).GT.XMAX) GO TO 160                                       SUP  189
0133            I=I+1                                                              SUP  190
0134            GOTO50                                                             SUP  191
              C                                                                    SUP  192
              C    THE FOLLOWING EQUATIONS ARE FOR A REGION 3 ANALYSIS.  THIS TYPE OF SUP  193
              C        ANALYSIS IS FOR THE POINTS ALONG THE CENTERLINE OF FLOW.  THE SUP  194
              C        NUMBERS FOR THESE POINTS CORRESPOND TO THE EQUATION:        SUP  195
              C                        I=(J+1)*NP+J                                SUP  196
              C                                                                    SUP  197
0135          120 VMM=V(I-1)                                                       SUP  198
0136            UMM=U(I-1)                                                         SUP  199
0137            DMM=D(I-1)                                                         SUP  200
0138            V(I)=0.0000                                                        SUP  201
0139            Y(I)=0.000                                                         SUP  202
0140            DX=100.                                                            SUP  203
0141          130 UTOP=-V(I-1)*(DMM-VMM**2/GRAV)-Y(I-1)*UMM*TAN(THETA)             SUP  204
0142            FMM=SQRT((UMM**2+VMM**2)/GRAV/DMM)                                 SUP  205
0143            UBOT=UMM*VMM/GRAV-DMM*SQRT(FMM**2-1.)                              SUP  206
0144            DELX=(-Y(I-1))*(DMM*GRAV-UMM**2)/(-UMM*VMM+DMM*GRAV*SQRT(FMM       SUP  207
                **2-1.))                                                           SUP  208
0145            X(I)=X(I-1)+DELX                                                   SUP  209
0146            U(I)=U(I-1)+UTOP/UBOT                                              SUP  210
0147            DELUN=U(I)-U(I-1)                                                  SUP  211
0148            DELVN=V(I)-V(I-1)                                                  SUP  212
0149            DELXN=X(I)-X(I-1)                                                  SUP  213
0150            DELYN=Y(I)-Y(I-1)                                                  SUP  214
0151            CALL DEPTH                                                         SUP  215
0152            IF(ABS(DX-D(I)).LE.ACC) GO TO 140                                  SUP  216
0153            VMM=(V(I)+V(I-1))/2.0                                              SUP  217
0154            UMM=(U(I)+U(I-1))/2.0                                              SUP  218
0155            DMM=(D(I)+D(I-1))/2.0                                              SUP  219
0156            DX=D(I)                                                            SUP  220
0157            GOTO130                                                            SUP  221
0158          140 JNPP=2*NP+1                                                      SUP  222
0159            IF(I.NE.JNPP) GO TO 150                                            SUP  223
0160            IF((X(JNPP)-X(NP)).GT.0.03) GO TO 150                              SUP  224
0161            GO TO 20                                                           SUP  225
0162          150 J=J+1                                                            SUP  226
0163            I=I+1                                                              SUP  227
0164            GOTO50                                                             SUP  228
0165          160 NR=I                                                             SUP  229
0166            I=0                                                                SUP  230
0167            QSTAR=2.*QSTAR                                                     SUP  231
0168            WRITE(6,1040) Q,QSTAR,SLOPE                                        SUP  232
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0169              WRITE(6,1050)                                                  SUP  233
0170              WRITE(6,1070)I,XO,YO,UO,VO,DO                                  SUP  234
0171              DO 170 I=1,NR                                                  SUP  235
0172            170 WRITE(6,1070)I,X(I),Y(I),U(I),V(I),D(I)                      SUP  236
0173              XP(1)=XO                                                       SUP  237
0174              XPT(1)=XO                                                      SUP  238
0175              DP(1)=DO                                                       SUP  239
0176              YPT(1)=YO                                                      SUP  240
0177              NRR=NR                                                         SUP  241
0178              DO 180 I=1,NRR                                                 SUP  242
0179              XPT(I+1)=X(I)                                                  SUP  243
0180            180 YPT(I+1)=Y(I)                                                SUP  244
0181              DO 190 I=1,J                                                   SUP  245
0182              XP(I+1)=X(I*NPP-1)                                             SUP  246
0183            190 DP(I+1)=D(I*NPP-1)                                           SUP  247
0184              K=J+1                                                          SUP  248
0185              KK=NR+1                                                        SUP  249
0186              XPT(KK+1)=MINXT                                                SUP  250
0187              XP(K+1)=MINXP                                                  SUP  251
0188              DP(K+1)=MINDP                                                  SUP  252
0189              YPT(KK+1)=MINYT                                                SUP  253
0190              EXXP(NEXP+1)=MINXP                                             SUP  254
0191              EXDP(NEXP+1)=MINDP                                             SUP  255
0192              XPT(KK+2)=SCXPT                                                SUP  256
0193              XP(K+2)=SCXP                                                   SUP  257
0194              DP(K+2)=SCDP                                                   SUP  258
0195              YPT(KK+2)=SCYPT                                                SUP  259
0196              EXXP(NEXP+2)=SCXP                                              SUP  260
0197              EXDP(NEXP+2)=SCDP                                              SUP  261
                C                                                                SUP  262
                C        SCALING OF THE VARIABLES TO BE PLOTTED                  SUP  263
                C                                                                SUP  264
0198              DO 200 I=1,NEXP                                                SUP  265
0199              EXXP(I)=(EXXP(I)-EXXP(NEXP+1))/EXXP(NEXP+2)                    SUP  266
0200            200 EXDP(I)=(EXDP(I)-EXDP(NEXP+1))/EXDP(NEXP+2)                  SUP  267
0201              DO 210 I=1,K                                                   SUP  268
0202              XP(I)=(XP(I)-XP(K+1))/XP(K+2)                                  SUP  269
0203            210 DP(I)=(DP(I)-DP(K+1))/DP(K+2)                                SUP  270
0204              DO 220 I=1,KK                                                  SUP  271
0205              XPT(I)=(XPT(I)-XPT(KK+1))/XPT(KK+2)                            SUP  272
0206            220 YPT(I)=(YPT(I)-YPT(KK+1))/YPT(KK+2)                          SUP  273
                C                                                                SUP  274
                C        LOCATING AND LABELING OF THE AXES                       SUP  275
                C                                                                SUP  276
0207              CALL AXIS (0.0,0.0,'DOWNSTREAM DISTANCE (FT)',-24,6.0,0.0,     SUP  277
                 (XP(K+1),XP(K+2))                                               SUP  278
0208              CALL AXIS (0.0,0.0,'DEPTH (FT)',10,3.,90.,DP(K+1),DP(K+2))     SUP  279
0209              CALL AXIS (0.0,5.0,'X',-1,6.0,0.0,XPT(KK+1),XPT(KK+2))         SUP  280
0210              CALL AXIS (0.0,5.0,'Y',1,3.0,90.,YPT(KK+1),YPT(KK+2))          SUP  281
0211              CALL PLOT(-1.0,5.0,3)                                          SUP  282
0212              CALL PLOT (6.0,5.0,2)                                          SUP  283
0213              CALL PLOT (6.0,7.0,3)                                          SUP  284
0214              CALL PLOT (0.0,0.0,2)                                          SUP  285
0215              CALL PLOT(-1.0,0.0,2)                                          SUP  286
                C                                                                SUP  287
                C        SUBROUTINE SMOOTH IS USED FOR THE ANALYTICAL RESULTS    SUP  288
                C                                                                SUP  289
                C                                                                SUP  290
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                C        PLOTS DEPTH VERSUS DOWNSTREAM DISTANCE                          SUP  291
                C                                                                        SUP  292
0216                     CALL SMOOTH (XP(1),DP(1),0)                                     SUP  293
0217                     DO 230 I=2,J                                                    SUP  294
0218              230 CALL SMOOTH(XP(I),DP(I),-2)                                        SUP  295
0219                     I=J+1                                                           SUP  296
0220                     CALL SMOOTH(XP(I),DP(I),-999)                                   SUP  297
0221                     CALL SYMBOL(4.0,3.0,0.11,'_ ANALYTICAL',0.0,12)                 SUP  298
0222                     CALL SYMBOL(4.0,2.75,.157,'  EXPERIMENTAL ',0.0,15)             SUP  299
0223                     CALL PLOT(0.0,0.0,-3)                                           SUP  300
                C                                                                        SUP  301
                C        SUBROUTINE SYMBOL IS USED FOR THE EXPERIMENTAL RESULTS          SUP  302
                C                                                                        SUP  303
0224                     DO 240 I=1,NEXP                                                 SUP  304
0225              240 CALL SYMBOL(EXXP(I),EXDP(I),0.08,1,0.0,-1)                         SUP  305
                C                                                                        SUP  306
                C        PLOTS FIRST -C CURVE                                            SUP  307
                C                                                                        SUP  308
0226                     CALL PLOT (0.0,5.0,-3)                                          SUP  309
0227                     CALL SMOOTH(XPT(1),YPT(1),0)                                    SUP  310
0228                     DO 250 I=2,NP                                                   SUP  311
0229              250 CALL SMOOTH(XPT(I),YPT(I),-2)                                      SUP  312
0230                     I=NP+1                                                          SUP  313
0231                     CALL SMOOTH(XPT(I),YPT(I),-999)                                 SUP  314
                C                                                                        SUP  315
                C        PLOTS SECOND -C CURVE                                           SUP  316
                C                                                                        SUP  317
0232                     I=NP+2                                                          SUP  318
0233                     CALL SMOOTH(XPT(I),YPT(I),0)                                    SUP  319
0234                     II=NP+3                                                         SUP  320
0235                     III=2*NP+1                                                      SUP  321
0236                     DO 260 I=II,III                                                 SUP  322
0237              260 CALL SMOOTH(XPT(I),YPT(I),-2)                                      SUP  323
0238                     I=2*NP+2                                                        SUP  324
0239                     CALL SMOOTH(XPT(I),YPT(I),-999)                                 SUP  325
                C                                                                        SUP  326
                C        PLOTS FIRST +C CURVE                                            SUP  327
                C                                                                        SUP  328
0240                     I=NP+1                                                          SUP  329
0241                     IF(XPT(I).GT.XMAX) GO TO 370                                    SUP  330
0242                     CALL SMOOTH(XPT(I),YPT(I),0)                                    SUP  331
0243                     II=2*NP+1                                                       SUP  332
0244                     III=NP*(NP+1)-NP+1                                              SUP  333
0245                     DO 270 I=II,III,NP                                              SUP  334
0246                     IF(I.GT.NR) GO TO 370                                           SUP  335
0247              270 CALL SMOOTH(XPT(I),YPT(I),-2)                                      SUP  336
0248                     I=NP*(NP+1)+1                                                   SUP  337
0249                     IF(I.GT.NR) GO TO 370                                           SUP  338
0250                     CALL SMOOTH(XPT(I),YPT(I),-999)                                 SUP  339
                C                                                                        SUP  340
                C        PLOTS SECOND +C CURVE                                           SUP  341
                C                                                                        SUP  342
0251                     I=2*NP+2                                                        SUP  343
0252                     IF(XPT(I).GT.XMAX) GO TO 370                                    SUP  344
0253                     CALL SMOOTH(XPT(I),YPT(I),0)                                    SUP  345
0254                     II=3*NP+2                                                       SUP  346
0255                     III=(NP+1)**2+1-NP                                              SUP  347
0256                     DO 280 I=II,III,NP                                              SUP  348
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0257              IF(I.GT.NR) GO TO 370                                  SUP  349
0258            280 CALL SMOOTH(XPT(I),YPT(I),-2)                        SUP  350
0259              I=(NP+1)**2+1                                          SUP  351
0260              IF(I.GT.NR) GO TO 370                                  SUP  352
0261              CALL SMOOTH(XPT(I),YPT(I),-999)                        SUP  353
                C                                                        SUP  354
                C     PLOTS THIRD -C CURVE                               SUP  355
                C                                                        SUP  356
0262              I=NP*(NP+1)+1                                          SUP  357
0263              IF(XPT(I).GT.XMAX) GO TO 370                           SUP  358
0264              CALL SMOOTH(XPT(I),YPT(I),0)                           SUP  359
0265              II=NP*(NP+1)+2                                         SUP  360
0266              III=(NP+1)*NP+NP                                       SUP  361
0267              DO 290 I=II,III                                        SUP  362
0268              IF(I.GT.NR) GO TO 370                                  SUP  363
0269            290 CALL SMOOTH(XPT(I),YPT(I),-2)                        SUP  364
0270              I=(NP+1)*NP+NP+1                                       SUP  365
0271              IF(I.GT.NR) GO TO 370                                  SUP  366
0272              CALL SMOOTH(XPT(I),YPT(I),-999)                        SUP  367
                C                                                        SUP  368
                C     PLOTS FOURTH -C CURVE                              SUP  369
                C                                                        SUP  370
0273              I=(NP+1)**2+1                                          SUP  371
0274              IF(XPT(I).GT.XMAX) GO TO 370                           SUP  372
0275              CALL SMOOTH(XPT(I),YPT(I),0)                           SUP  373
0276              II=(NP+1)**2+2                                         SUP  374
0277              III=(NP+2)*NP+NP+1                                     SUP  375
0278              DO 300 I=II,III                                        SUP  376
0279              IF(I.GT.NR) GO TO 370                                  SUP  377
0280            300 CALL SMOOTH(XPT(I),YPT(I),-2)                        SUP  378
0281              I=(NP+2)*NP+NP+2                                       SUP  379
0282              IF(I.GT.NR) GO TO 370                                  SUP  380
0283              CALL SMOOTH(XPT(I),YPT(I),-999)                        SUP  381
                C                                                        SUP  382
                C     PLOTS THIRD +C CURVE                               SUP  383
                C                                                        SUP  384
0284              I=(NP+2)*NP+1                                          SUP  385
0285              IF(XPT(I).GT.XMAX) GO TO 370                           SUP  386
0286              CALL SMOOTH(XPT(I),YPT(I),0)                           SUP  387
0287              II=(NP+2)*NP+NP+1                                      SUP  388
0288              III=NPP*2*NP-NP+1                                      SUP  389
0289              DO 310 I=II,III,NP                                     SUP  390
0290              IF(I.GT.NR) GO TO 370                                  SUP  391
0291            310 CALL SMOOTH(XPT(I),YPT(I),-2)                        SUP  392
0292              I=NPP*2*NP+1                                           SUP  393
0293              IF(I.GT.NR) GO TO 370                                  SUP  394
0294              CALL SMOOTH(XPT(I),YPT(I),-999)                        SUP  395
                C                                                        SUP  396
                C     PLOTS FOURTH +C CURVE                              SUP  397
                C                                                        SUP  398
0295              I=(NP+3)*NP+2                                          SUP  399
0296              IF(XPT(I).GT.XMAX) GO TO 370                           SUP  400
0297              CALL SMOOTH(XPT(I),YPT(I),0)                           SUP  401
0298              II=(NP+3)*NP+NP+2                                      SUP  402
0299              III=(NP+1)*(2*NP+1)-NP+1                               SUP  403
0300              DO 320 I=II,III,NP                                     SUP  404
0301              IF(I.GT.NR) GO TO 370                                  SUP  405
0302            320 CALL SMOOTH(XPT(I),YPT(I),-2)                        SUP  406
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0303              I=(NP+1)*(2*NP+1)+1                                    SUP  407
0304              IF(I.GT.NR) GO TO 370                                  SUP  408
0305              CALL SMOOTH(XPT(I),YPT(I),-999)                        SUP  409
                C                                                        SUP  410
                C         PLOTS FIFTH -C CURVE                           SUP  411
                C                                                        SUP  412
0306              I=2*NP*(NP+1)+1                                        SUP  413
0307              IF(XPT(I).GT.XMAX) GO TO 370                           SUP  414
0308              CALL SMOOTH(XPT(I),YPT(I),0)                           SUP  415
0309              II=2*NP*(NP+1)+2                                       SUP  416
0310              III=(2*NP+3)*NP                                        SUP  417
0311              DO 330 I=II,III                                        SUP  418
0312              IF(I.GT.NR) GO TO 370                                  SUP  419
0313          330 CALL SMOOTH(XPT(I),YPT(I),-2)                          SUP  420
0314              I=(2*NP+3)*NP+1                                        SUP  421
0315              IF(I.GT.NP) GO TO 370                                  SUP  422
0316              CALL SMOOTH(XPT(I),YPT(I),-999)                        SUP  423
                C                                                        SUP  424
                C         PLOTS SIXTH -C CURVE                           SUP  425
                C                                                        SUP  426
0317              I=(2*NP+1)*(NP+1)+1                                    SUP  427
0318              IF(XPT(I).GT.XMAX) GO TO 370                           SUP  428
0319              CALL SMOOTH(XPT(I),YPT(I),0)                           SUP  429
0320              II=(2*NP+1)*(NP+1)+2                                   SUP  430
0321              III=2*(NP+2)*NP+1                                      SUP  431
0322              DO 340 I=II,III                                        SUP  432
0323              IF(I.GT.NR) GO TO 370                                  SUP  433
0324          340 CALL SMOOTH(XPT(I),YPT(I),-2)                          SUP  434
0325              I=2*(NP+2)*NP+2                                        SUP  435
0326              IF(I.GT.NR) GO TO 370                                  SUP  436
0327              CALL SMOOTH(XPT(I),YPT(I),-999)                        SUP  437
                C                                                        SUP  438
                C         PLOTS FIFTH +C CURVE                           SUP  439
                C                                                        SUP  440
0328              I=(2*NP+3)*NP+1                                        SUP  441
0329              IF(XPT(I).GT.XMAX) GO TO 370                           SUP  442
0330              CALL SMOOTH(XPT(I),YPT(I),0)                           SUP  443
0331              II=(2*NP+3)*NP+NP+1                                    SUP  444
0332              III=(2*NP+3)*NP+1+(NP-1)*NP                            SUP  445
0333              DO 350 I=II,III,NP                                     SUP  446
0334              IF(I.GT.NR) GO TO 370                                  SUP  447
0335          350 CALL SMOOTH(XPT(I),YPT(I),-2)                          SUP  448
0336              I=(2*NP+3)*NP+1+(NP-1)*NP                              SUP  449
0337              IF(I.GT.NR) GO TO 370                                  SUP  450
0338              CALL SMOOTH(XPT(I),YPT(I),-999)                        SUP  451
                C                                                        SUP  452
                C         PLOTS SIXTH +C CURVE                           SUP  453
                C                                                        SUP  454
0339              I=2*(NP+2)*NP+2                                        SUP  455
0340              IF(XPT(I).GT.XMAX) GO TO 370                           SUP  456
0341              CALL SMOOTH(XPT(I),YPT(I),0)                           SUP  457
0342              II=2*(NP+2)*NP+NP+2                                    SUP  458
0343              III=2*(NP+2)*NP+2+(NP-1)*NP                            SUP  459
0344              DO 360 I=II,III,NP                                     SUP  460
0345              IF(I.GT.NR) GO TO 370                                  SUP  461
0346          360 CALL SMOOTH(XPT(I),YPT(I),-2)                          SUP  462
0347              I=2*(NP+2)*NP+(NP-1)*NP+NP+2                           SUP  463
0348              IF(I.GT.NR) GO TO 370                                  SUP  464
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0349              CALL SMOOTH(XPT(I),YPT(I),-999)                                  SUP  465
                C                                                                  SUP  466
                C     SHIFTS PLOTTER PEN FOR ANOTHER SET OF DATA                   SUP  467
                C                                                                  SUP  468
0350            370 CONTINUE                                                       SUP  469
0351              CALL PLOT (20.0,0.0,-3)                                          SUP  470
0352            380 CONTINUE                                                       SUP  471
0353              CALL PLOT (0.0,0.0,999)                                          SUP  472
0354           1000 FORMAT(I5)                                                     SUP  473
0355           1010 FORMAT(8F8.4)                                                  SUP  474
0356           1020 FORMAT(8F10.5)                                                 SUP  475
0357           1030 FORMAT(I5,2F10.5)                                              SUP  476
0358           1040 FORMAT('1',///////////////,22X,'MAIN CHANNEL FLOWRATE:',F7.4,   SUP  477
                   ' CFS',5X,'LATERAL INFLOW:',F8.5,' CFS/FT',4X,'SLOPE:',F5.2,    SUP  478
                   ' (%)                                                           SUP  479
0359           1050 FORMAT(' ',25X,'POINT',5X,'DOWNSTREAM',6X,'TRANSVERSE',12X,    SUP  480
                   ('VELOCITY (FPS)',9X,'DEPTH',/,36X,'DISTANCE (FT)',3X,'DISTANCE (FT)SUP  481
                   (',4X,'X-DIRECTION',2X,'Y-DIRECTION',5X,'(FT)')                 SUP  482
0360           1060 FORMAT(2F10.5)                                                 SUP  483
0201           1070 FORMAT(' ',23X,I5,4X,F6.3,10X,F6.3,10X,F7.4,7X,F7.4,5X,F7.4)   SUP  484
0362              RETURN                                                           SUP  485
0363              END                                                              SUP  486
```

```
0001              SUBROUTINE DEPTH                                                DPH    1
0002              COMMON U(500),D(500),V(500),X(500),Y(500),I,NP,NPP,J,THETA,     DPH    2
                 (ALPHA,UMM,UNP,VMM,VNP,DMM,DNP,DELUN,DELUP,DELVN,DELVP,DELXN,    DPH    3
                 (DELXP,DELYN,DELYP,DELX,DELYO,GRAV,DELU,UO,QSTAR,YO,XO,FNP,Q1,UO DPH    4
0003              II=(J+1)*NP+J                                                   DPH    5
0004              IF(I.NE.II) GO TO 30                                            DPH    6
0005           10 G11=ATAN(ABS(V(I-1))/ABS(U(I-1)))                               DPH    7
0006              G21=ATAN(ABS(DELYN)/ABS(DELXN))                                 DPH    8
0007              G1=ATAN(ABS(V(I))/ABS(U(I)))                                    DPH    9
0008              G2=G21                                                          DPH   10
0009              D=G21+G11                                                       DPH   11
0010              IF(V(I-1).LT.0.0) D=G21-G11                                     DPH   12
0011              P=G2+G1                                                         DPH   13
0012              IF(V(I).LT.0.0) P=G2-G1                                         DPH   14
0013              VP1=SQRT(U(I-1)**2+V(I-1)**2)*COS(D)                            DPH   15
0014              VP=SQRT(U(I)**2+V(I)**2)*COS(P)                                 DPH   16
0015              B=2.*VP**2/GRAV-TAN(THETA)*COS(G21)*SQRT(DELXN**2+DELYN**2)     DPH   17
0016              C=(-2.*VP1**2/GRAV-TAN(THETA)*COS(G21)*SQRT(DELXN**2+DELYN      DPH   18
                 (**2)-D(I-1))*D(I-1)                                            DPH   19
0017              GO TO 40                                                        DPH   20
0018           20 G21=ATAN(ABS(DELYO)/ABS(DELX))                                  DPH   21
0019              VP1=UO*COS(G21)                                                 DPH   22
0020              G2=G21                                                          DPH   23
0021              VP=U(I)*COS(G2)                                                 DPH   24
0022              B=2.*VP**2/GRAV-TAN(THETA)*COS(G2)*SQRT(DELX**2+DELYO**2)       DPH   25
0023              C=(-2.*VP1**2/GRAV-TAN(THETA)*COS(G2)*SQRT(DELX**2+DELYO**2)    DPH   26
                 (-UO)*UO                                                        DPH   27
0024              GO TO 40                                                        DPH   28
0025           30 IF(I.EQ.1) GO TO 20                                             DPH   29
0026              IF(I.LE.NP) GO TO 10                                            DPH   30
0027              G15=ATAN(ABS(V(I-NP))/ABS(U(I-NP)))                             DPH   31
0028              G25=ATAN(ABS(DELYP)/ABS(DELXP))                                 DPH   32
0029              G1=ATAN(ABS(V(I))/ABS(U(I)))                                    DPH   33
0030              G2=G25                                                          DPH   34
0031              C=G25-G15                                                       DPH   35
0032              IF(V(I-NP).LT.0.0) D=G25+G15                                    DPH   36
0033              P=G2-G1                                                         DPH   37
0034              IF(V(I).LT.0.0) P=G1+G2                                         DPH   38
0035              VP5=SQRT(U(I-NP)**2+V(I-NP)**2)*COS(D)                          DPH   39
0036              VP=SQRT(U(I)**2+V(I)**2)*COS(P)                                 DPH   40
0037              B=2.*VP**2/GRAV-TAN(THETA)*COS(G2)*SQRT(DELXP**2+DELYP**2)      DPH   41
0038              C=(-2.*VP5**2/GRAV-TAN(THETA)*COS(G2)*SQRT(DELXP**2+DELYP**2)   DPH   42
                 (-D(I-NP))*D(I-NP)                                              DPH   43
0039           40 D(I)=(-B+SQRT(B**2-4.*C))/2.                                    DPH   44
0040              RETURN                                                          DPH   45
0041              END                                                             DPH   46
```

```
0001              SUBROUTINE DYDX                                                DYDX   1
0002              COMMON U(500),D(500),V(500),X(500),Y(500),I,NP,NPP,J,THETA,    DYDX   2
                 (ALPHA,UMM,UNP,VMM,VNP,DMM,DNP,DELUN,DELUP,DELVN,DELVP,DELXN,   DYDX   3
                 (DELXP,DELYN,DELYP,DELX,DELYO,GRAV,DELU,DD,QSTAR,YO,XO,FNP,Q1,UO DYDX   4
0003              IF(I.EQ.NPP) GO TO 10                                          DYDX   5
0004              Q1=QSTAR*ABS(X(I-NP)-X(I-NPP))/COS(ALPHA)                      DYDX   6
0005              GO TO 20                                                       DYDX   7
0006           10 Q1=QSTAR*ABS(X(I-NP)-XO)/COS(ALPHA)                            DYDX   8
0007           20 CONTINUE                                                       DYDX   9
0008              ZZ=Q1*Q1*VNP**2/16./DNP**4/GRAV**2/DY**2+Q1*UNP/2./DNP**2/GRAV/DY DYDX  10
0009              Z=(Y(I-NP)-YO)/TAN(ALPHA)-X(I-NP)                              DYDX   11
0010              AAAA=-UNP*VNP*Z-GRAV*DNP*Z*SQRT(FNP**2.-1.+ZZ)+Q1*VNP/4./DNP/TAN(ADYDX  12
                 (LPHA)                                                         DYDX   13
0011              BBBB=.1*VNP*Z/4./DNP                                           DYDX   14
0012              CCCC=-UNP*VNP/TAN(ALPHA)-GRAV*DNP/TAN(ALPHA)*SQRT(FNP**2.-1.+ZZ) DYDX  15
0013              DDDD=UNP*GRAV-UNP*UNP                                          DYDX   16
0014              EEEE=-Q1*UNP/2./DNP                                            DYDX   17
0015              BAC=(AAAA-EEEE)**2.+BBBB*(DDDD-CCCC)                           DYDX   18
0016              IF (BAC.LT.0.0) GO TO 30                                       DYDX   19
0017              DY=(AAAA-EEEE+SQRT(BAC))/2./(DDDD-CCCC)                        DYDX   20
0018              GO TO 40                                                       DYDX   21
0019           30 DELX=1000.                                                     DYDX   22
0020              WRITE(6,1000)                                                  DYDX   23
0021         1000 FORMAT(' ',/////21X,'DELTA Y FAILS TO CONVERGE; THE CHARACTERISTICDYDX  24
                 ( METHOD CAN NOT BE APPLIED TO THIS SET OF DATA')              DYDX   25
0022           40 Y(I)=Y(I-NP)+DY                                               DYDX   26
0023              X(I)=(Y(I)-YO)/TAN(ALPHA)                                      DYDX   27
0024              RETURN                                                         DYDX   28
0025              END                                                            DYDX   29
```